Màster en Mètodes Numèrics

Computational Structural Mechanics and Dynamics

Assignment 7: Plates

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a) Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate. Use the 5x5 element mesh.

$$\text{Material properties: } \begin{cases} E = 10.92 \\ \nu = 0.3 \\ Q = 1.0 \\ t = 0.001; 0.01; 0.02; 0.1; 0.4 \end{cases}$$

The simulations are carried out for the 2 elements, Reissner Mindlin and MZC, with different thicknesses, with a 5x5 element mesh and the material properitess described above. On all edges strong Simple Support Uniform boundary conditions are imposed with uniform load. The length of each side is 4.

A good way to compare is the Normalized displacement, *Figure 2.* In this figure, there are observable similarities between Reissner Mindlin plate theory and Timoshenko beam theory while there also exists equivalence between MZC plate theory and Euler Bernoulli beam theory. This is one reason why for MZC element no information about shear stress is obtained; because that model assumes the shear deformation as negligible.

Reissner Mindlin works well for thick plates. However, for thin plates it suffers from transverse shear locking effect and gives stiffer results compared to MZC model theory (which would be preferred in thin models). On the other case, RM element plate theory provides information about shear forces, but due to the existence of shear locking effects on thin elements, the need for using a reduced integration for the shear matrix is expected.

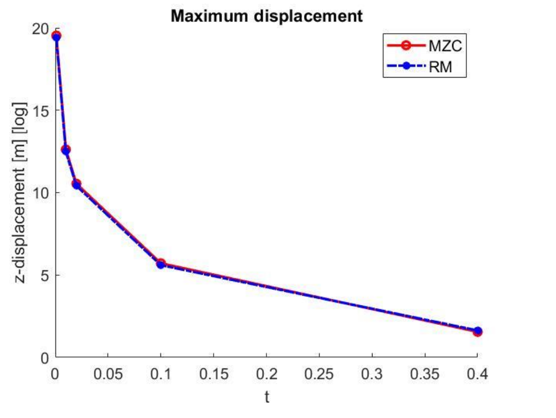

Figure 1. Maximum z-displacement.

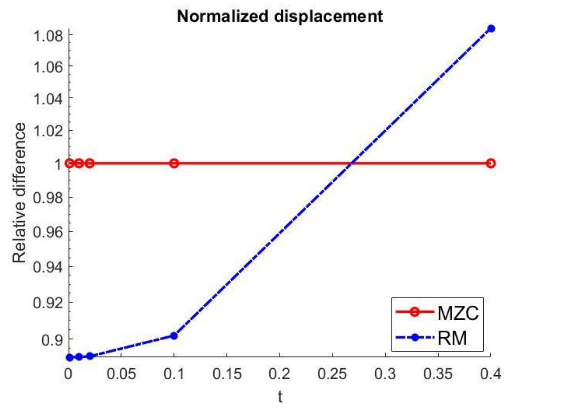

Figure 2. Normalized displacement.

The results shown below are the z-component displacements for both element models and shear stress in case of RM model in a plate whose thickness is 0.001.

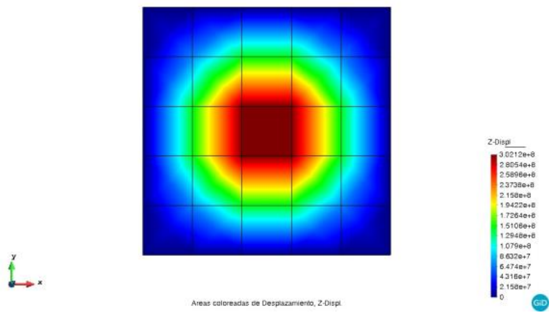

Figure 3. MZC z-displacement.

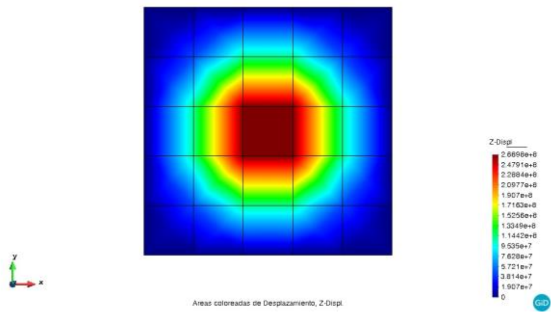

Figure 4. RM z-displacement.

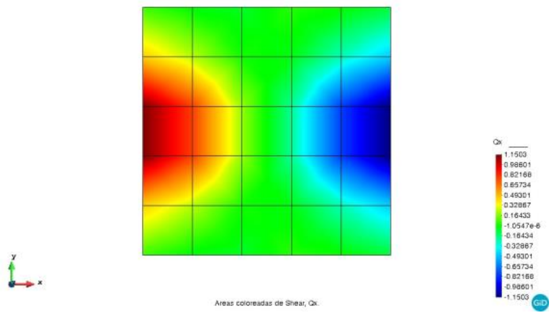

Figure 5. RM Qx-shear stress.

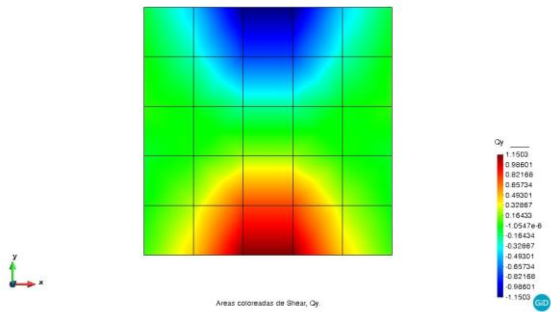

Figure 6. RM Qy-shear stress.

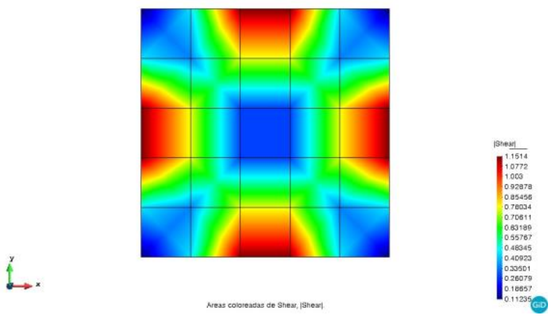

Figure 7. RM shear stress.

b) Define and verify a patch test mesh for the MZC element.

In this exercise, Zienkiewicz element is tested using two meshes; one with regular elements and the second one has distorted elements. The material parameters are defined as:

$$\text{Material properties: } \begin{cases} E = 10.92 \\ \nu = 0.3 \\ \rho = 0 \\ t = 0.001 \end{cases}$$

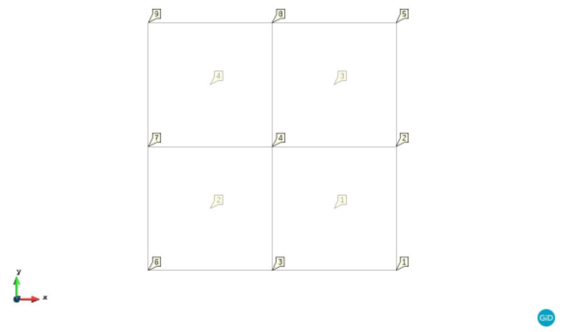

Figure 8. MZC regular element mesh.

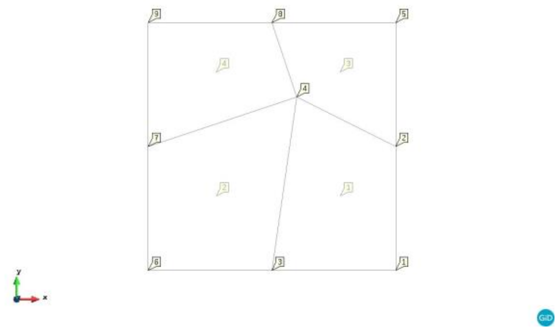

Figure 9. MZC non-regular element mesh.

A first test is done imposing a constant solution, *Figure 10,* on the boundary nodes so as to check that the solution in the inner node is fulfilled. On the *Figure 11,* it is imposed a linear solution on the boundary of the regular mesh, $u_z = x + 2y + 3$; It is seen the element preserves the linear solution.

x-coordinate	**y-coordinate**	**Imposed constant displacement** ($\boldsymbol{\theta_x} = \mathbf{0}; \boldsymbol{\theta_y} = \mathbf{0}$)	**Imposed linear displacement** ($\boldsymbol{\theta_x} = \mathbf{1}; \boldsymbol{\theta_y} = \mathbf{2}$)
1	0	0	4
1	0.5	0	5
0.5	0	0	3.5
0.5	0.5	0	4.5
1	1	0	6
0	0	0	3
0	0.5	0	4
0.5	1	0	5.5
0	1	0	5

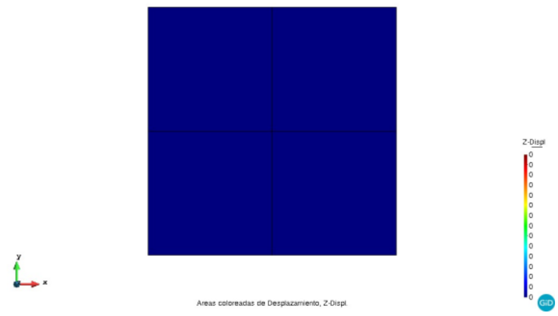

Figure 10. MZC z-displacement constant solution in regular element mesh.

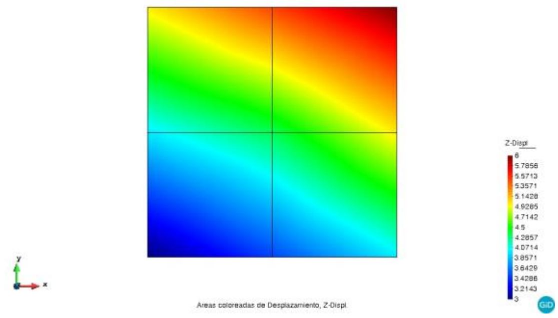

Figure 11. MZC z-displacement linear solution in regular element mesh.

On the other hand, the same test is performed on a non-orthogonal element mesh. First imposing the same constant solution in the boundary, *Figure 12,* and then imposing the linear solution in the non-regular element mesh, *Figure 13.*

x-coordinate	**y-coordinate**	**Imposed constant displacement** ($\boldsymbol{\theta_x = 0; \theta_y = 0}$)	**Imposed linear displacement** ($\boldsymbol{\theta_x = 1; \theta_y = 2}$)
1	0	0	4
1	0.5	0	5
0.5	0	0	3.5
0.6	0.7	0	5
1	1	0	6
0	0	0	3
0	0.5	0	4
0.5	1	0	5.5
0	1	0	5

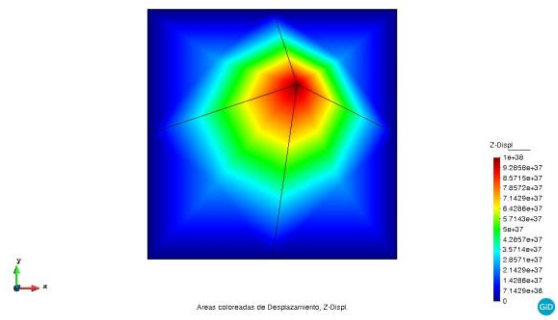

Figure 12. MZC z-displacement constant solution in non-regular element mesh.

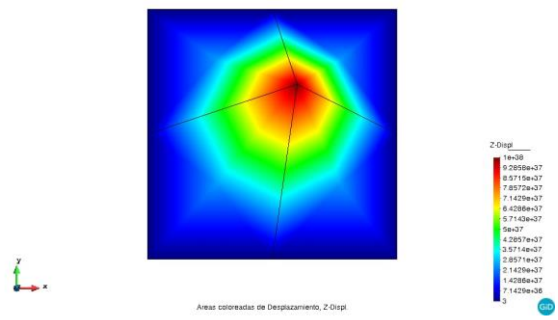

Figure 13. MZC z-displacement linear solution in non-regular element mesh.

It is seen that the Zienkiewicz element passes the patch test only when the elements are not distorted. The element is suitable just when orthogonal meshes are used (rectangular, squared)