Assignment 7: Plates

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Introduction

Many 3D structures can be simplified to 2D structures when two of the three dimensions are much bigger than the other one and when the geometry of our problem is plane. In this case, the plate theory can be applied, which simplifies the computational calculus. This is not the only advantage, the plate theory is well known and studied so their results are reliable and easy to interpret.
In the following study, two theories will be studied: Melosh-Zienkiewicz-Cheung theory (MZC) and Reissner-Mindlin theory (RM).

Results

In order to compare both theories, the same problem has been solved, it is, 5x5 mesh with same mechanical properties and a uniformly distributed force Q=1.
For both theories, the distribution of the physical magnitudes look similar with some differences in the value of the magnitude. As it can be seen in *Figure 1.* the displacements have the same distribution, however, the absolute value is different depending on the theory.

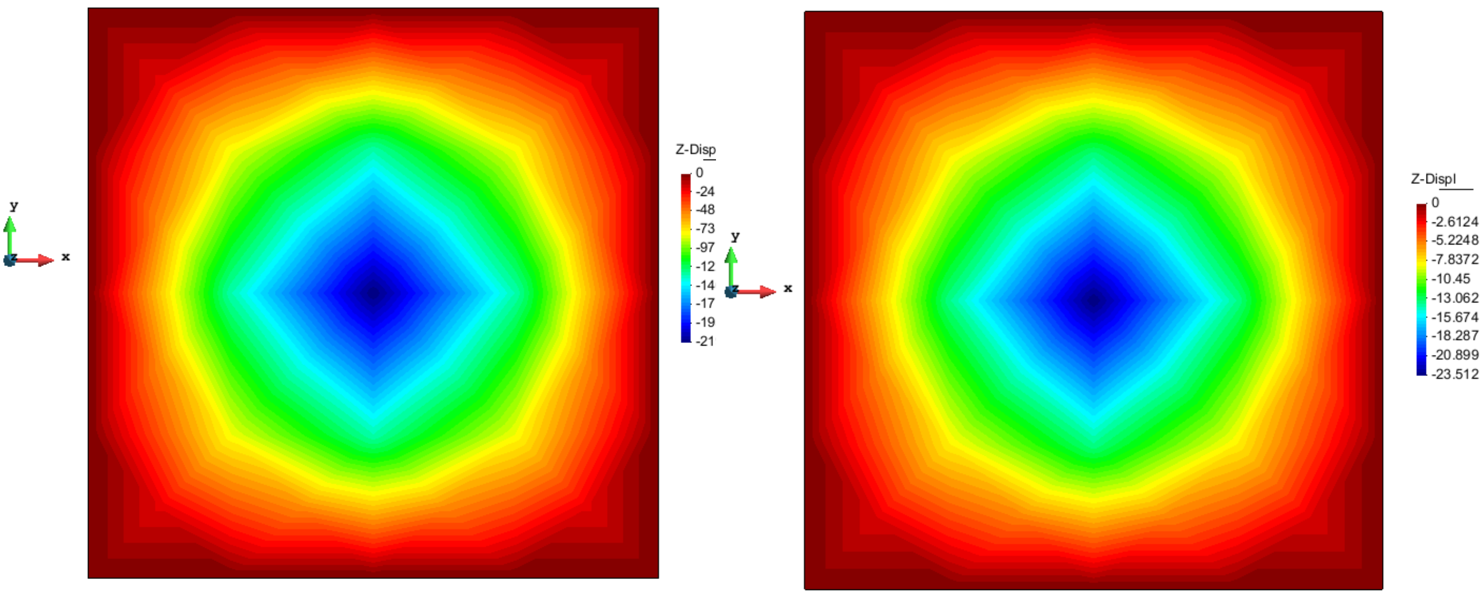

Figure 1. Representation of the displacements for MZC elements (left) and RM elements (right).

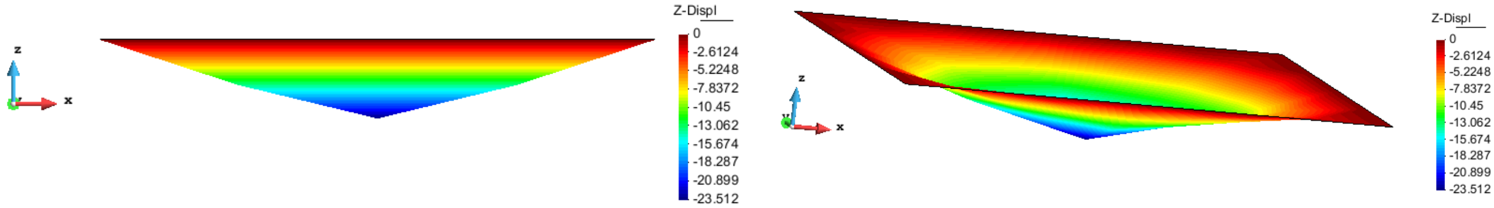

Figure 2. Deformed shape of the plate for RM elements with different perspectives.

Given that for both theories the deformed shape is the same but the only change is the magnitude of the displacements only the deformed shape of RM elements have been plotted. The only difference between theories is the magnitude of the displacements, not the distribution and the shape.

Comparing the displacement of the middle point for different thickness of the plate, the following results are obtained by changing only the thickness of the plate:

	MZC	RM
Thickness	u_z	u_z
0,001	-1,41E+10	-10174
0,01	-1,41E+07	-1017,3
0,02	-1,76E+06	-508,6
0,1	-14077	-101,21
0,4	-219,95	-23,512

Table 1. Displacement for the middle point for different thickness.

As it can be seen in *Table 1*. the displacement for the middle point is different depending on the theories and it becomes bigger and bigger when the thickness is reduced. This is due to shear-locking effect of the RM theory. When the thickness is reduced, the stiffness matrix have different contributions: bending stiffness and shear stiffness. Given that the first one vanishes faster than the second one, when thickness is really small the total stiffness will be dominated by the shear stiffness. This shear stiffness has no physical sense and produces the plate to be more 'rigid' and therefore the displacements are not as big as in MZC theory.
This fact has also been seen when studying beams, when comparing Euler-Bernouilli theory and Timoshenko theory the same shear-locking effect raised. In this case, for plates we have the same problem because MZC theory is equivalent to Euler-Bernouilli and RM theory is equivalent to Timoshenko beam theory.

Regarding moments and rotation, the distribution of those magnitudes have been plotted only for one type of element, given that the distribution along the plate is the same but with different magnitudes due to the displacements magnitude is different but the distribution is the same.

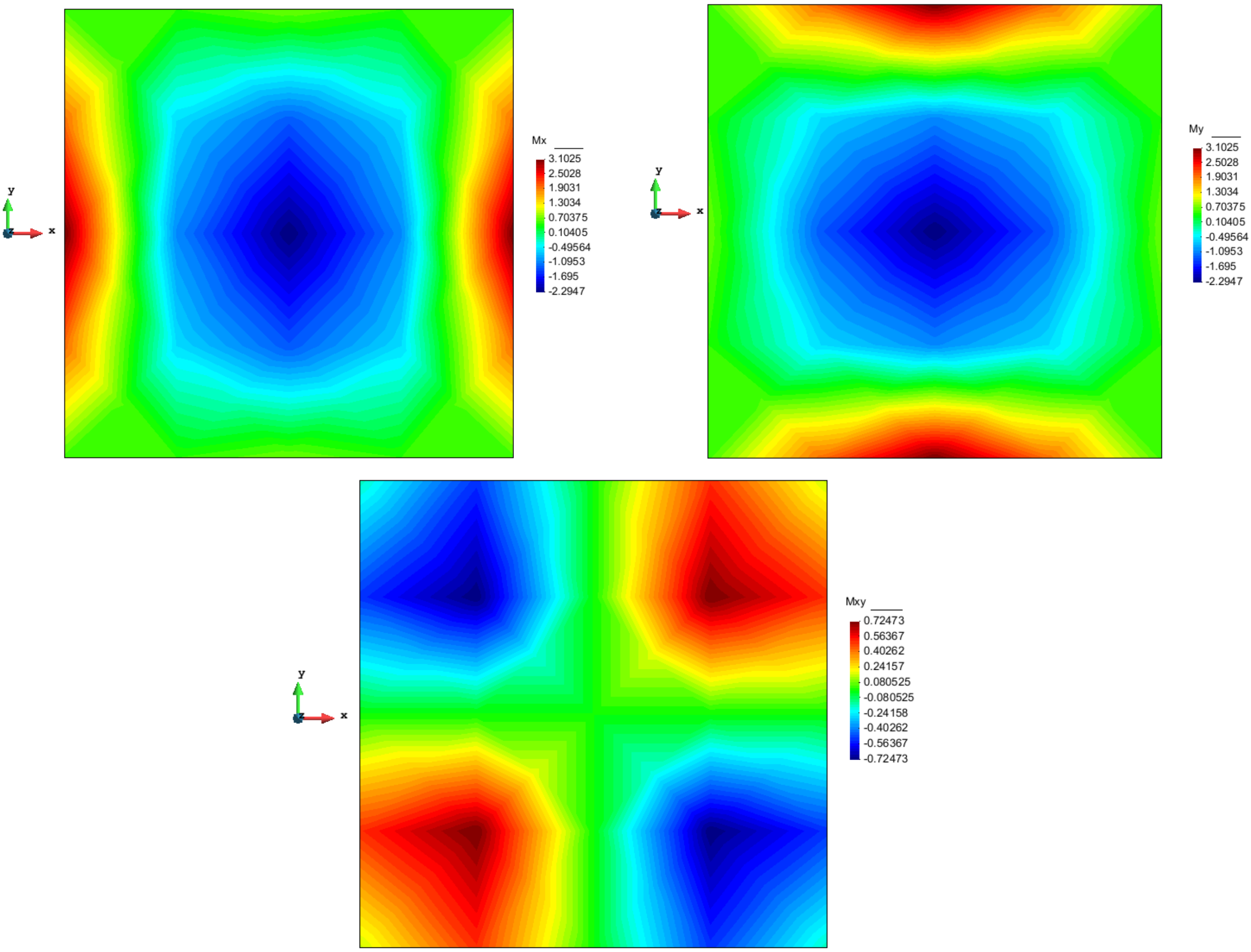

Figure 3. Representation of the moment in The x direction (left), in the y direction (right) and Mxy (bottom) for MZC elements.

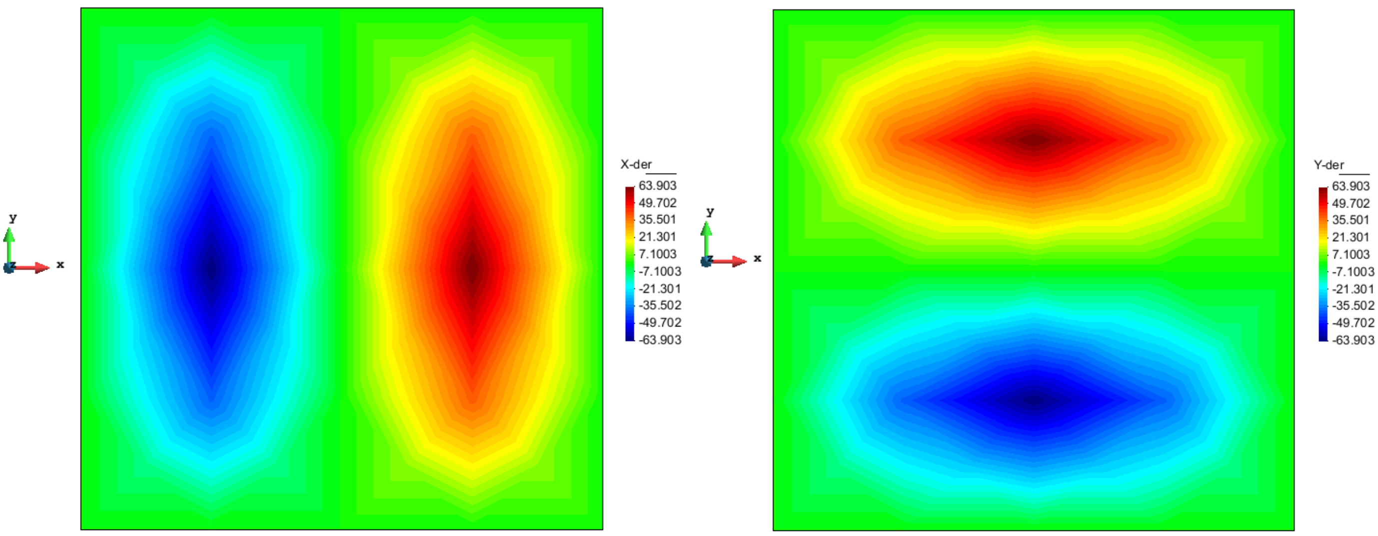

Figure 4. Representation of the rotation in the x direction(left) and in the y direction (right) for MZC elements.

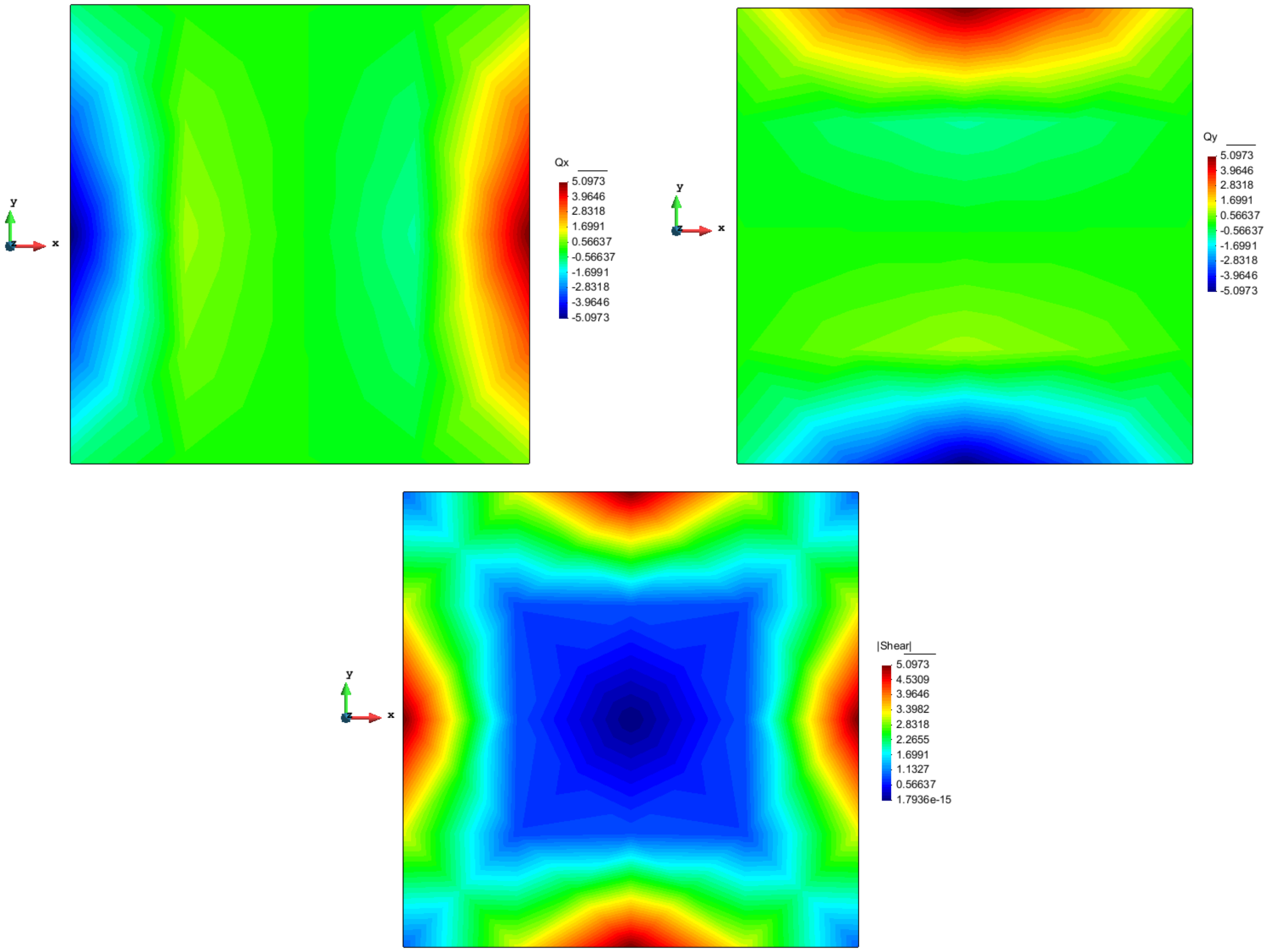

Figure 5. Representation of the shear forces in the x direction (left), in the y direction (right) and modulus of the shear forces (bottom) for the RM elements.

It must be remarked that, even though moments and rotations have only been plotted for MZC elements and not for RM elements because the visual representation looks the same, shear forces only have sense for RM elements. The reason for that is the same as in beam theory. MZC theory does not take into account shear effects and RM does. So only for RM elements the distribution of shear forces can be represented.

To end this work, a patch test for MZC elements have been studied. The proposed patch test is the following:

For a 3x3 mesh, the displacements in the boundary nodes have been imposed by setting u_z=0 for the left face u_z=20 for the right face and u_z=10 for the mid points in the boundary. With that, a linear displacement field has been imposed in the plate. When Q=0, the MZC theory with linear elements should interpolate the displacement field according to this imposed linear field. If it does interpolate exactly this field, then MZC satisfies the patch test.

As it can be seen in the following figure, MZC satisfies the patch test.

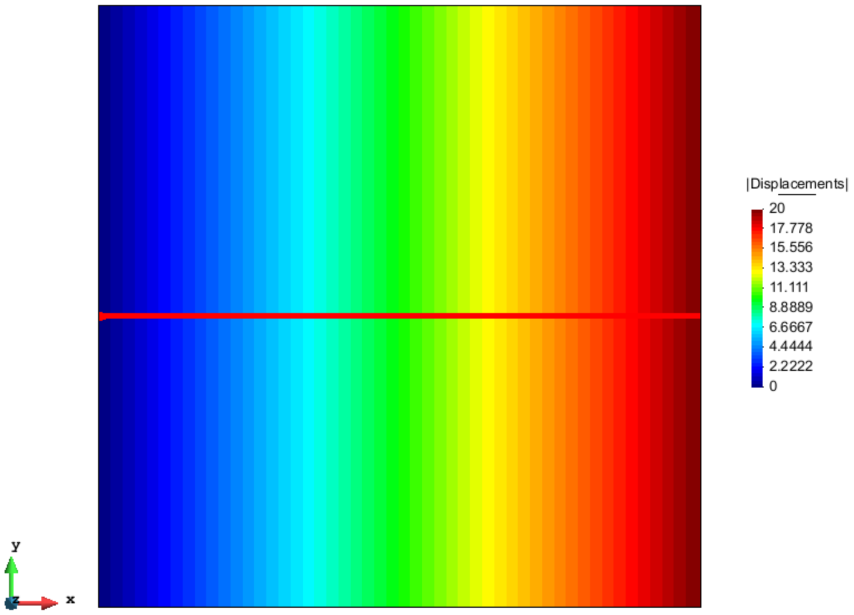

Figure 6. Representation of the displacement for the proposed patch test with a cut line (red line) in the x direction.

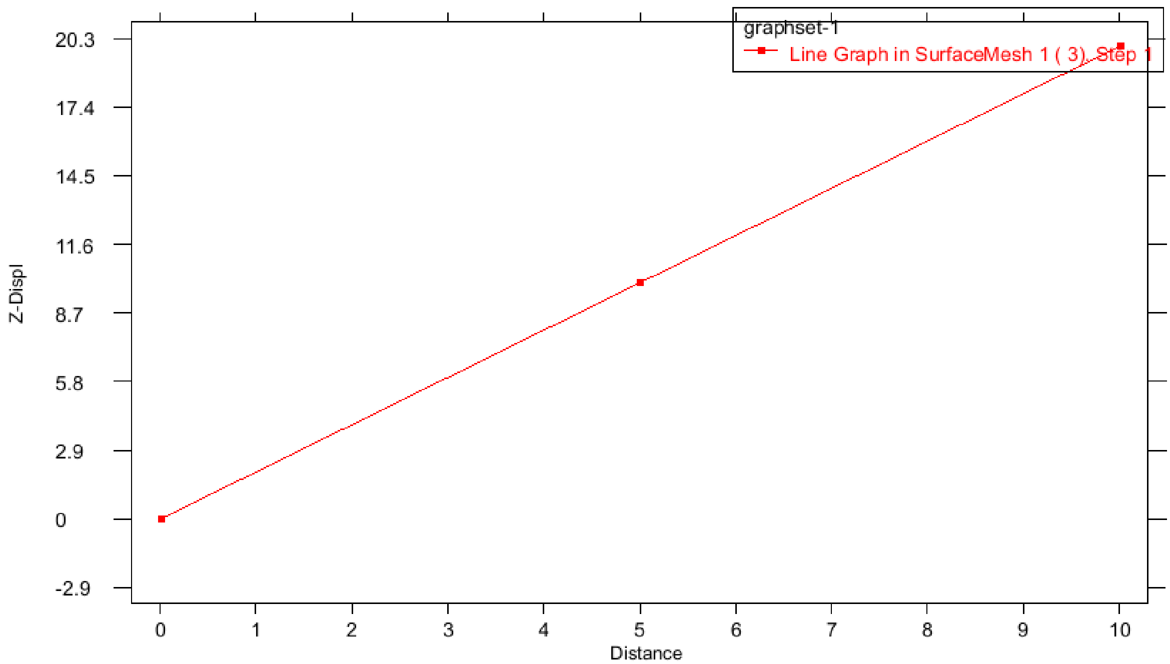

Figure 7. Displacement in the z direction in terms of the distance in the cut line.

From *Figure 6.* the visual representation of the displacements shows that the displacement are in the x direction and remain constant for a fixed y. *Figure 7.* shows that the displacement in the cut line (set in the x direction) are linear. Given that the imposed displacement field of the boundary is exactly interpolated for MZC elements, it is proved that they satisfies the patch test.