

CSMD, Plates Theory

Assignment 7:

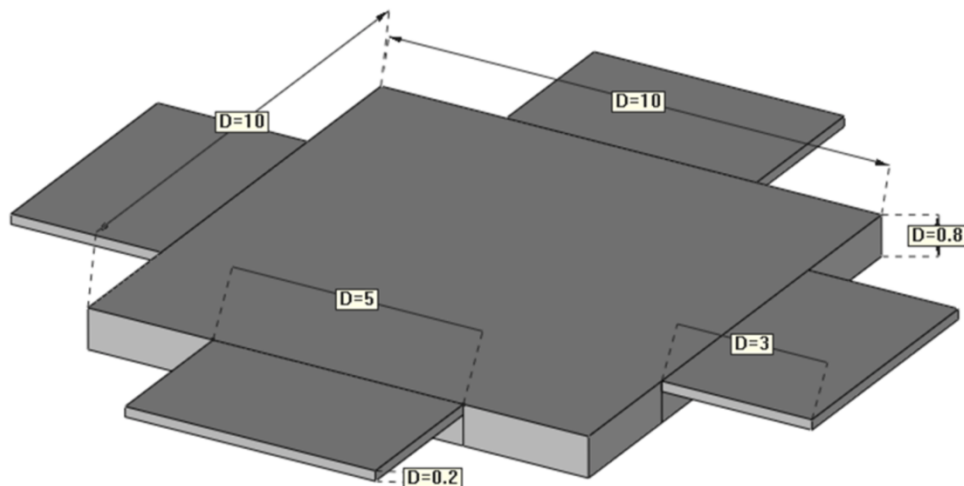
- a) What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problems:

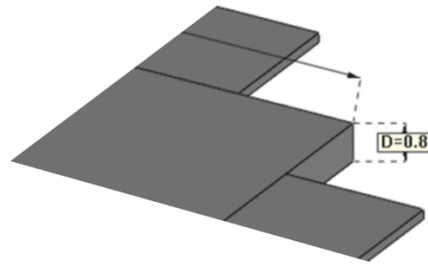
The first approach for this type of structure it is to find a solution/deal with non-match plates middle plane. In order to keep a good solution that it converges to the original problem it can be suggested to introduce a dummy element in order to connect both elements ($10 \times 10 \times 0.8$) and ($3 \times 5 \times 0.2$).

The introduction of this dummy element it will help to cover the jump between both elements and run a good solution for the problem. Another important feature that the dummy element has to keep as a property it is to be enough stiff in order to not take any energy and just transfer all the information (displacement, rotations, strain, stress, etc.).

If we move now to answer which theory it can be applied to solve the structure, one can think direct on thin plate theory due to the relation of the elements (wide and thick), but indeed it can be applied both theories (thin and thick). Just it would be convenient check the desirable accuracy on the shear information or result from the structure. As it is well-known, for thin plates the main energy is taken by the full bending matrix meanwhile for thick elements, the full shear matrix can turn to lack information and corresponding errors on the bending matrix introducing shear lock effect.

Due to the symmetry of the geometry, we can consider the $\frac{1}{4}$ of the total structure and simplify the problem.

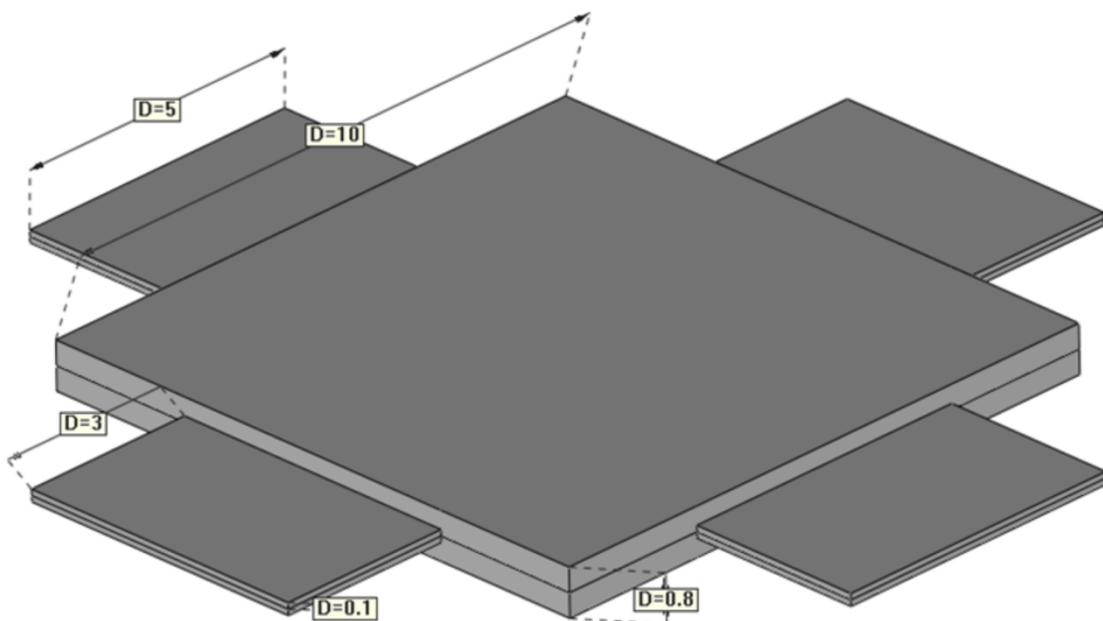




b) What kind of strategy (theory, elements, integration rule, boundary conditions, Etc.) will you use for solving the following problems:

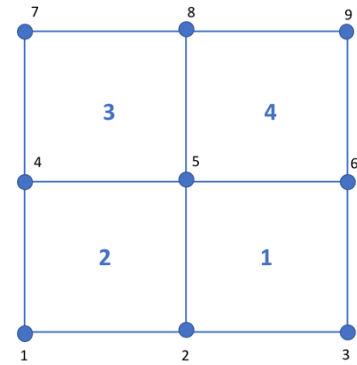
In this case, we find a structure composed of different plates thickness, but with the main difference that all of them match on their middle plane. That easy way allows us the possibility to model it as it is, and furthermore, use for all of them the same theory for thin elements.

The model criteria would be, create a structure mesh where we can define the different thickness of the elements. In the case that shear forces are important and could be not taken properly by the thin plate theory; in the jumps (change of the elements thickness) it would be convenient the refinement of the mesh and apply the *Reissner-Midlin* theory.



c) Define and verify a patch test mesh for the MCZ element.

The patch test for the MCZ plate it will consist on the take a number of nodes (select the patch) and prescribed a displacement for the exterior of nodes of the patch (chosen). Furthermore, the internal forces have to be prescribed to be zero and keep constant the strains (as the displacements). On the other side, to ensure that the FEM code pass the patch test, the displacement of the internal patch node had to be exactly as the prescribed external nodes displacement and keep constant the strains.



In the patch test, the node 5 it is expected to move the same quantity as the prescribed displacement on the external nodes.

The displacements functions are the following:

$$w = \frac{1}{2}(x^2 + y^2 + xy)$$

$$\theta_x = x + \frac{1}{2}y$$

$$\theta_y = y + \frac{1}{2}x$$

On the next table, we find different displacement datum for each node:

N.ID	X coord	Y coord	W	θ_x	θ_y
1	0	0	0	0	0
2	0.5	0	0.125	0.5	0.25
3	1	0	0.5	1	0.5
4	0	0.5	0.125	0.25	0.5
5	0.5	0.5	0.375	0.75	0.75
6	1	0.5	0.875	1.25	1
7	1	0	0.5	0.5	1
8	0.5	1	0.875	1	1.25
9	1	1	1	1.5	1.5

The results for the node 5, comparison between the hand solution and the FEM solution is shown on the following table.

	W	θ_x	θ_y
Analytic	0.3750	0.750	0.749
FEM	0.3751	0.7493	0.750