Universitat Politecnica De Catalunya, Barcelona
Masters in Computational Mechanics

Course
Computational Structural Mechanics and Dynamics

Assignment on 'Plates'

By
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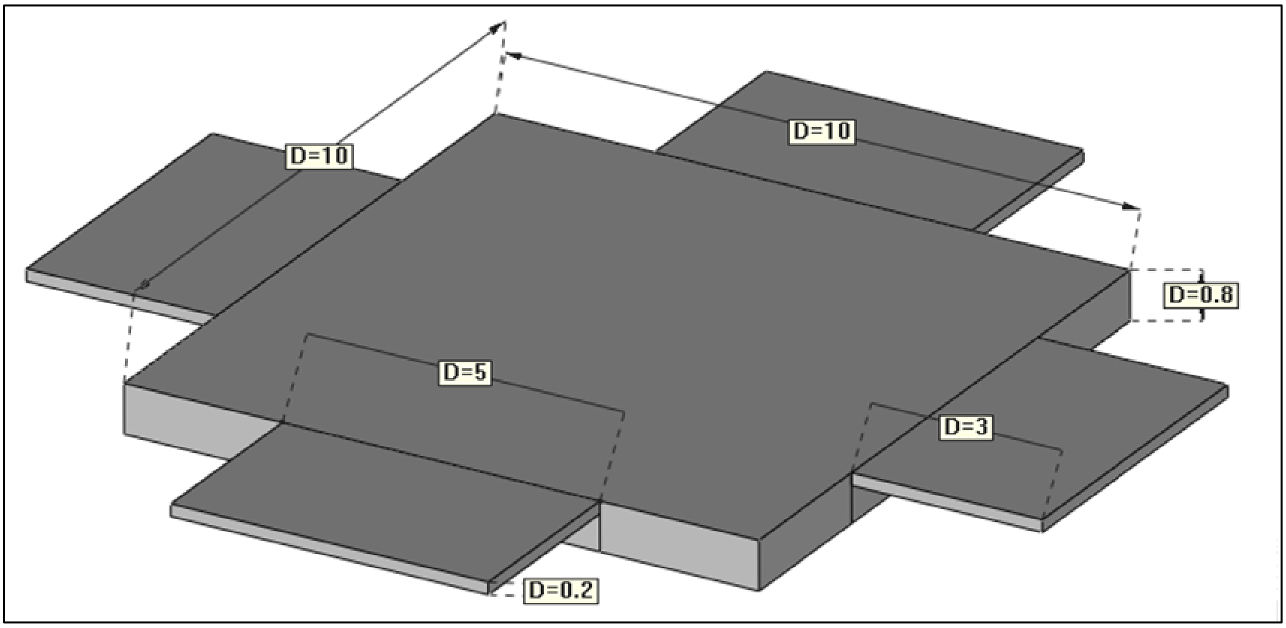
Ques 1. (A) Think first and answer later. What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problems?



Solution:

a. **Theory: Reissner-Mindlin Plate Theory**
   We consider share deformation, as to the top surface of the main plates, edge plates are attached. Independent of the position of the mid surface, plate can rotate.

b. **Element: Rectangular Element**
   Pure bending represented by 9 node Lagrange rectangular element. Since the vertical displacement have quadratic variation and pure bending is resulted rather than zero bending by zero shear deformation. Each element poses 27 degrees of freedom.

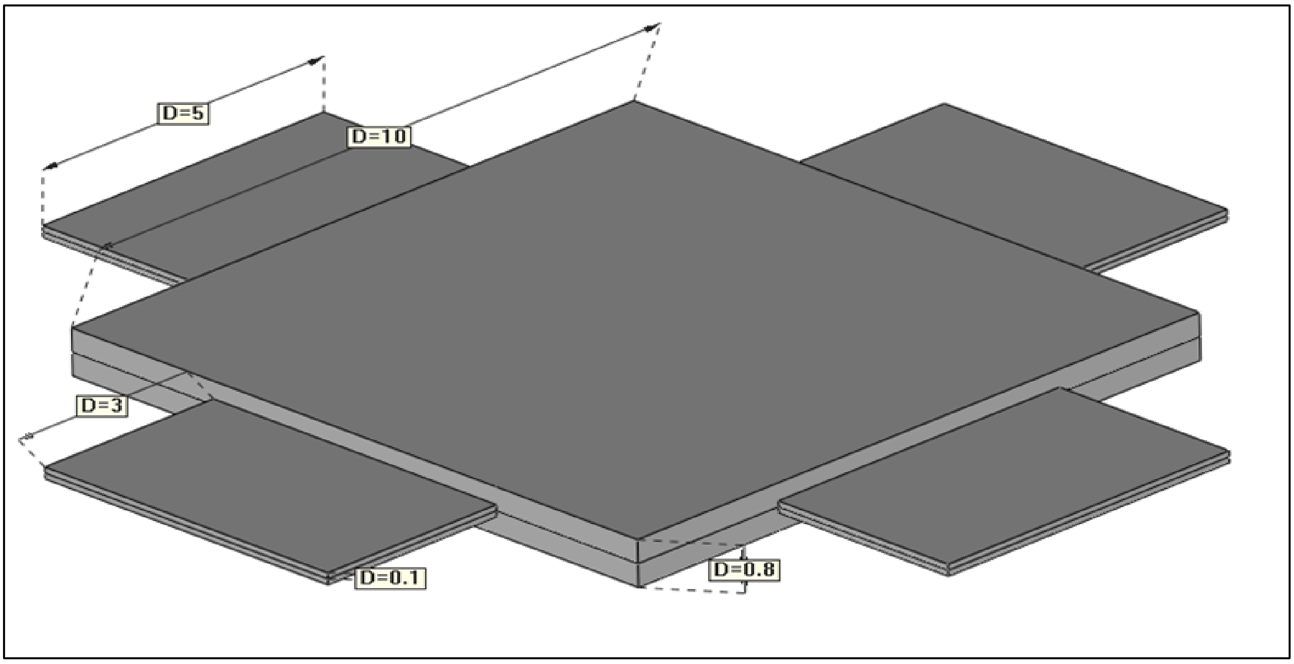
c. **Integration: Reduced Integration**
   The ratio of thickness to characteristic length is considered for integration. The ratio is 8/100 for main plate (10*0.8) & 2/50 for edge plates (5*0.2). The reduced integration is more accurate with low computational cost.

d. **Boundary Conditions:**
   Force is applied on the main plate at the middle and the edge plates are either clamped or simply supported at their outer edge, Boundary conditions are assumed accordingly

Clamped Condition: w = θ_x = θ_y = 0
Simply Supported Condition: w = θ_x = 0 or w = θ_y = 0
Weak Supported Condition: w = 0

where, w is vertical displacement andθ_x&θ_y are first order partial derivative of w

Ques 1. (B) Think first and answer later. What kind of strategy (theory, elements, integration rule, boundary conditions, etc.) will you use for solving the following problems?



Solution:

a. **Theory: Kirchhoff Plate Theory**
   The plate can't rotate independently of the position of the mid surface since the thin plates are attached to the main plate at the middle. Shear Deformation (transverse shear stress) is zero.

b. **Element: Rectangular Element**
   In consideration of its properties such as accuracy, low computational cost and perfectly compatibles with the geometry, **BFS** element of rectangular shape is selected

c. **Integration: Full-exact Integration**
   For integration, the ratio of thickness to characteristic length is considered, the ratio is 8/100 for main plate (10*0.8) & 2/50 for edge plates (5*0.2). The stiffness matrix is not a major issue, so Full Integration is used.

d. **Boundary Conditions:**
   Force is applied on the main plate at the middle and the edge plates are either clamped or simply supported at their outer edge, Boundary conditions are assumed accordingly

Clamped Condition: w = θ_x = θ_y = 0

Simply Supported Condition: w = θ_x = 0 or w = θ_y = 0

Weak Supported Condition: w = 0

where, w is vertical displacement andθ_x&θ_y are first order partial derivative of w.

Ques 2. Define and verify a patch test mesh for the MZC element?

Solution:

Non-conforming rectangular Melosh-Zienkiewicz-Cheung (MZC) element:

1. Approximation for w (vertical displacement):

$$w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3$$

2. Arbitrary values of α (alpha):

α	Value
1	0.004
2	0.004032
3	0.004065
4	0.004098
5	0.004131
6	0.004165

α	Value
7	0.0042
8	0.004235
9	0.004271
10	0.004308
11	0.004345
12	0.004382

In GID the patch of the element is created, then above stated equation is solved. The displacement values obtained are assigned to origin, edge section, and in GID the middle section values are cross verified with post processing displacement value.

3. Geometry:

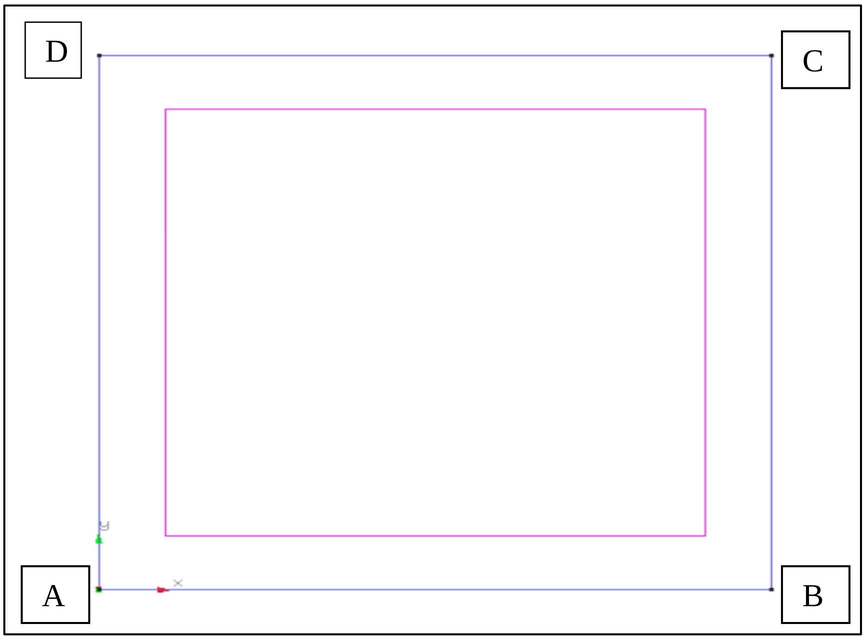
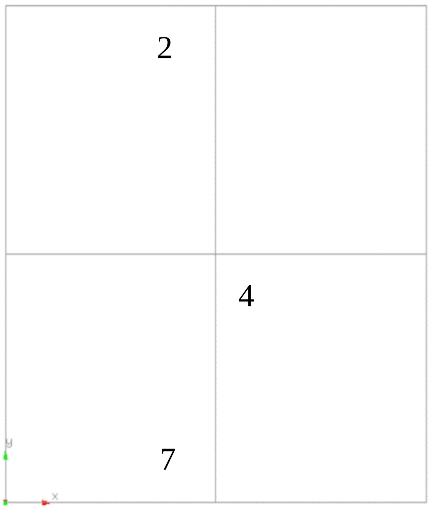
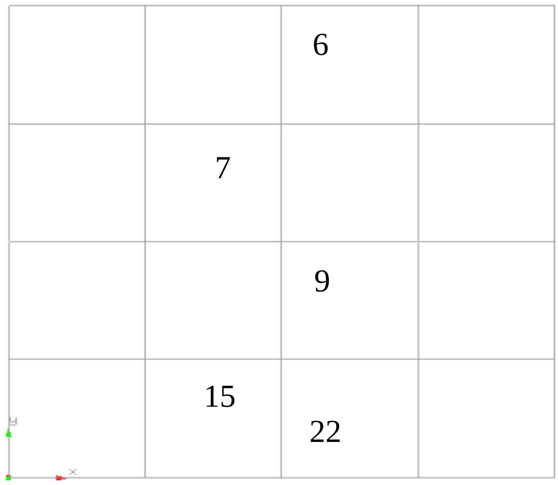


Fig: Plate Geometry

Coordinates: A (0,0); B (4,0); C (4,4); D (0,4).

4. Mesh Geometry:

	
4 Element (9 nodes)	16 Element (25 nodes)

5. Post Processing (Displacement)

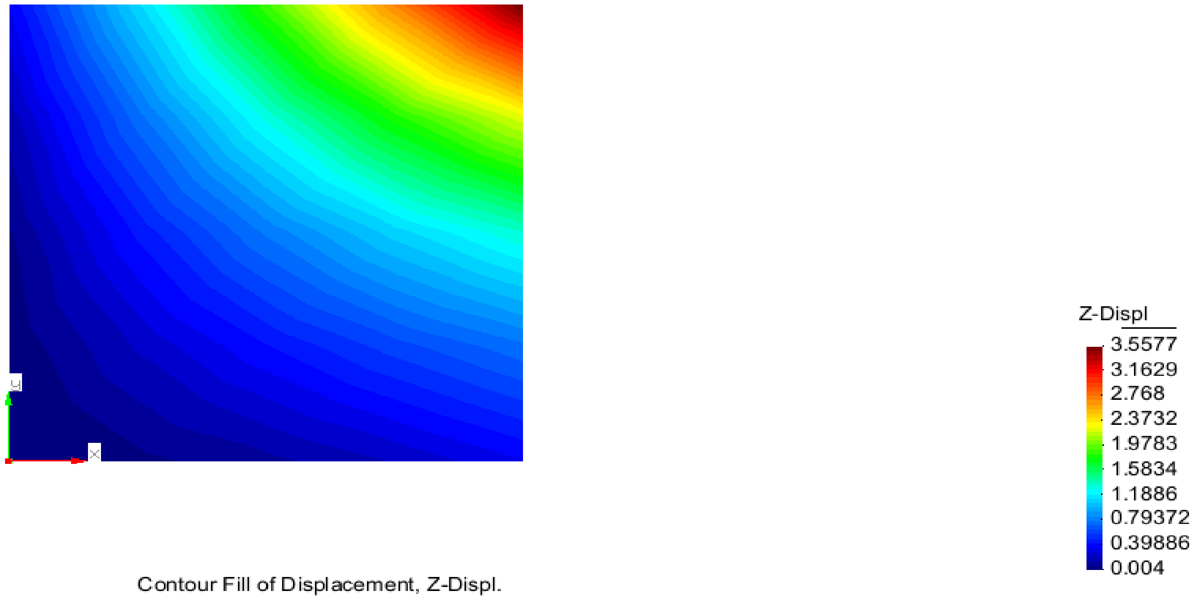


Fig: Z displacement

6. Explicit Data:

For 4 Elements

	x	y	x2	y2	x3	y3	w
Mid	2	0	4	0	8	0	0.062054
	2	2	4	4	8	8	0.345515
	2	4	4	16	8	64	1.358119

Edge	4	0	16	0	64	0	0.354487
	4	2	16	4	64	8	1.347015
	4	4	16	16	64	64	3.557741
Origin	0	0	0	0	0	0	0.004
	0	2	0	4	0	8	0.063251
	0	4	0	16	0	64	0.362589

For 16 Elements

	x	**y**	**x2**	**y2**	**x3**	**y3**	**w**
Mid	2	0	4	0	8	0	0.062054
	2	1	4	1	8	1	0.151859
	2	2	4	4	8	8	0.345515
	2	3	4	9	8	27	0.721457
	2	4	4	16	8	64	1.358119
Edge	4	0	16	0	64	0	0.354487
	4	1	16	1	64	1	0.763989
	4	2	16	4	64	8	1.347015
	4	3	16	9	64	27	2.234591
	4	4	16	16	64	64	3.557741
Origin	0	0	0	0	0	0	0.004
	0	1	0	1	0	1	0.016537
	0	2	0	4	0	8	0.063251
	0	3	0	9	0	27	0.169986
	0	4	0	16	0	64	0.362589

7. **Error Calculation:** Error is calculated at Path nodes as:

$$\text{Err} = \text{Abs}[\frac{(\text{Cal w} - \text{Obs w})}{\text{Cal w}}]$$

For 4 Element:

Patch Node	**Calculated w**	**Observed w**	**Error**
2	1.358119	1.90E+00	3.99E-01
4	0.345515	7.99E-01	1.31E+00
7	0.062054	1.24E-01	9.98E-01

For 16 Element:

Patch Node	**Calculated w**	**Observed w**	**Error**
6	1.358119	1.89E+00	3.95E-01
7	0.721457	1.30E+00	7.96E-01
9	0.345515	7.96E-01	1.30E+00
15	0.151859	4.22E-01	1.78E+00
22	0.062054	1.22E-01	9.65E-01

Conclusion:

As we can witness that the error reduces during the error calculation as the mesh is refined, which indicates the convergence of the result. Hence the MZC element satisfies the patch test can be concluded.

-------x-------x-------x-------