Course:
Computational Structural Mechanics and Dynamics

Assignment-7

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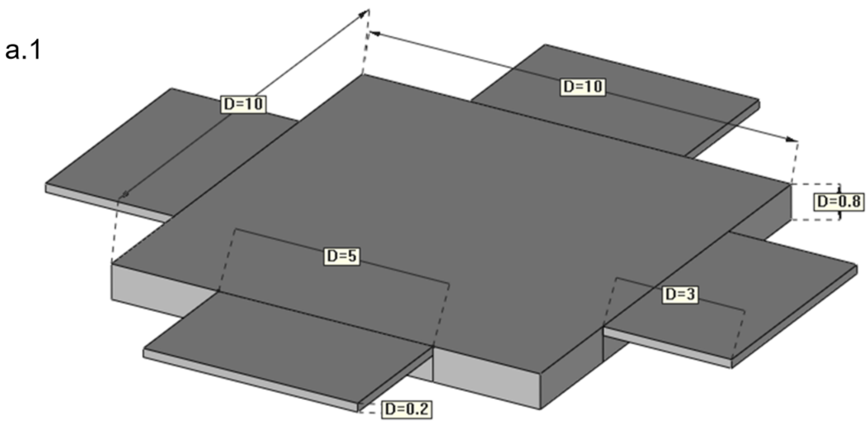
Exercise 1

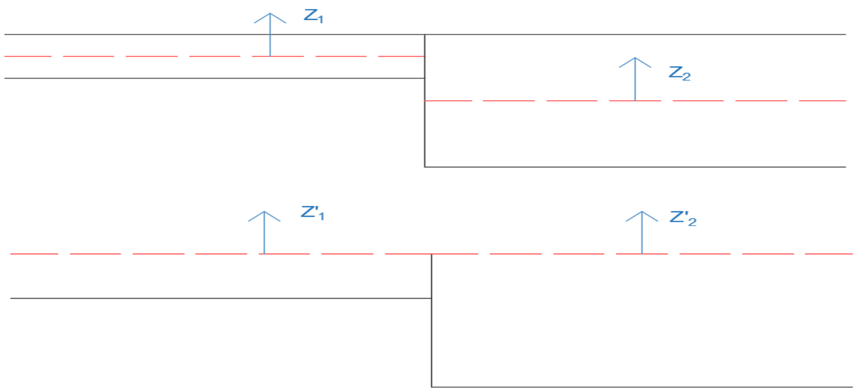
What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problems:

Problem 1}



The first structure shows the problem of offset between the elements (the plates don't share the same middle plane). Two solutions can be used to model this structure.

The first solution is to focus on each element, referring to its natural middle plane. Then, during the assembly process of the global stiffness matrix, the reference systems are moved to the top edge of the elements. These edges become the new "middle plane" for the structure.



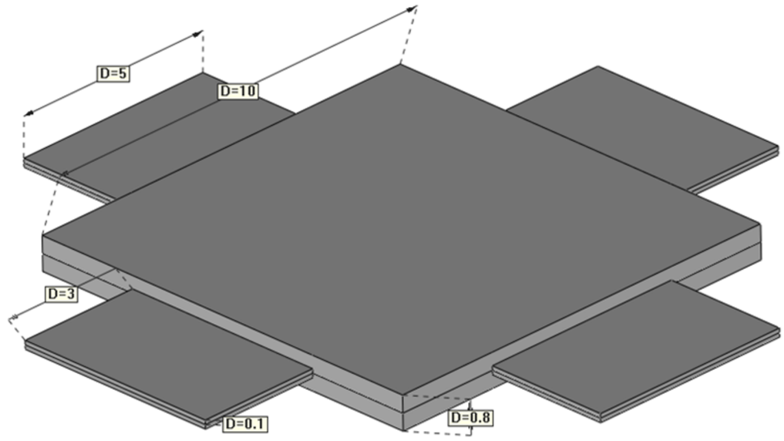
The second possibility is to use a rigid element that connects the two offset middle planes. In this case, the modeling process is more complex but has a more specific physical meaning.

Both theories (Kirchhoff's and Mindlin/Reissner's) can be used for this problem since the plates are all thin (the central plate dimensional ratio is equal to 0.08 while the maximum ratio for the small plates is equal to 0.067, both less than 0.1). If the Mindlin/Reissner's theory is used, it must be taken into account the possible problem of shear locking, in particular if the main deformation problem is for bending. In this case the integration rule consists in the reduction of the shear stiffness matrix K_s of the plates while the bending

stiffness K_b follow a full integration process.
In any case, it's recommended to use Mindlin/Reissner's elements close to the jumps of thickness.
Since the structure shows two axis of symmetry, it's possible to divide the problem into four parts (corners) and focus only on one quarter. It's necessary to add boundary conditions along the edges of cut: along the edge parallel to the axis *x*, the rotation θ_y must be equal to 0, while along the edge parallel to the axis *y*, the rotation $\theta_x = 0$.

Problem 2}

a.2



The second structure doesn't show problems about offset of the middle planes. The structure shows two axis of symmetry, it's possible to divide the problem into four parts (corners) and focus only on one quarter. It's necessary to add boundary conditions along the edges of cut: along the edge parallel to the axis *x*, the rotation θ_y must be equal to 0, while along the edge parallel to the axis *y*, the rotation $\theta_x = 0$.
For the same geometrical reasons shown in the first part of the paragraph (here the maximum ratio of the small plates is 0.033), in this problem both theories can be used too. While considering the whole structure as one single "plate" element, the ratio is still less than 0.1 (0.05) and both theories can be used.
If the shear problem is predominant, the thin/thick plates theory (Mindlin/Reissner's theory) is necessary, taking into account the possible shear locking effect: in this case it's important to calculate a reduced version of the shear stiffness matrix K_s and leaving full integrated the bending K_b. If the main deformation is bending a Kirchhoff's theory elements can be used. In any case, it's recommended to use Mindlin/Reissner's elements close to the jumps of thickness.

Exercise 2

Define and verify a patch test mesh for the MCZ element:

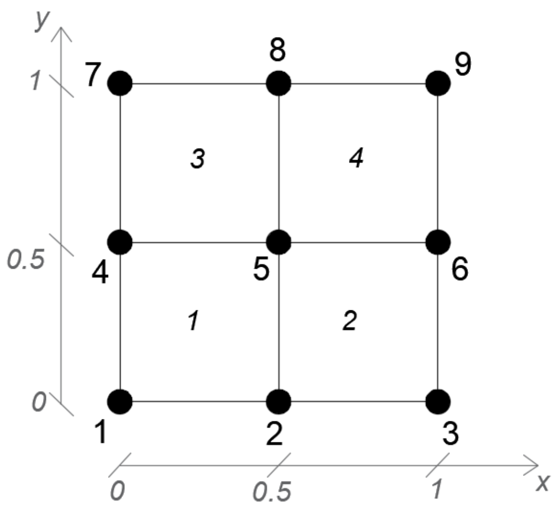
In this part a patch test for the displacements is implemented, using MCZ elements. The patch test follows this structure: firstly, a set of displacement functions are assumed.

$$w = \frac{1}{2}(x^2 + y^2 + xy)$$

$$\theta_x = x + \frac{1}{2}y$$

$$\theta_y = \frac{1}{2}x + y$$

Secondly, the displacements of each node of the plate structure are calculated analytically, using the coordinates of the nodes.



Node	X	Y	w	θ_x	θ_y
1	0	0	0	0	0
2	0.5	0	0.125	0.5	0.25
3	1	0	0.5	1	0.5
4	0	0.5	0.125	0.25	0.5
5	0.5	0.5	0.375	0.75	0.75
6	1	0.5	0.875	1.25	1
7	0	1	0.5	0.5	1
8	0.5	1	0.875	1	1.25
9	1	1	1	1.5	1.5

Then, prescribing the displacements of the exterior nodes (just calculated analytically), and assuming the internal forces equal to 0, a FEM code which follows the MCZ hypothesis is implemented and the displacements of the internal node are calculated as unknowns. The patch test is satisfied if the displacements of the internal node match with the analytical ones. Moreover, the strain field must be constant through the whole plate element.

Since the relations for the strains are:

$$\varepsilon_x = -\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = -\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = -\frac{\partial^2 w}{\partial xy}$$

The field for the strains is constant. The results of the patch test are the following:

Node	Calculation	w	θ_x	θ_y	ε_x	ε_y	γ_{xy}
5	Analytical	0.375	0.75	0.75	-1	-1	-1
	FEM Code	0.3751	0.7493	0.7500	-1.0036	-0.9976	-1.0000

Since the results are very similar to the analytical calculation, the structure passed the patch test.