Universitat Politècnica de Catalunya

Computational Solid Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Shells

Author:
Pau Márquez

Supervisor:
Prof. F. Zárate

Academic Year 2019-2020

Contents

1 Stresses on an hyperbolic-paraboloid shell

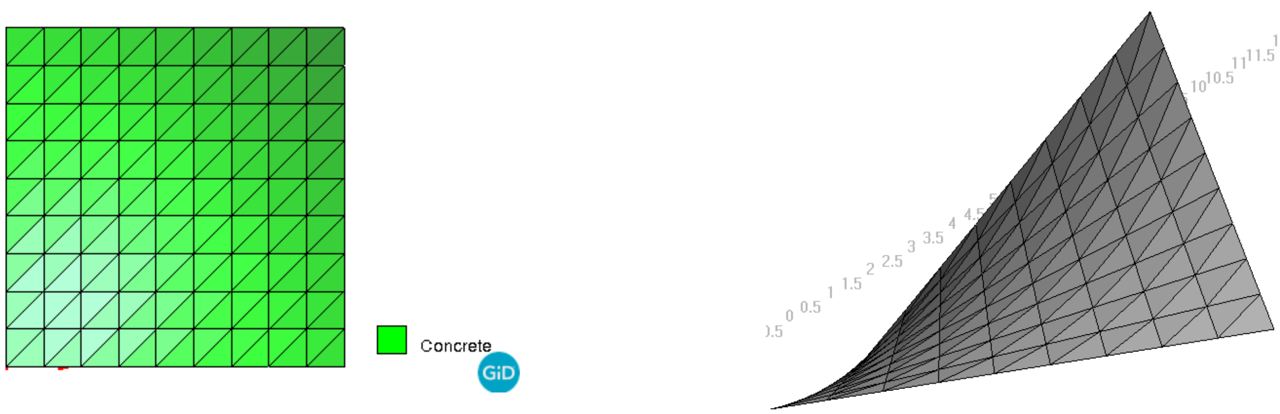
The shell that we are asked to analyze is an hyperbolic-paraboloid characterized by a concave and a convex parabola. In order to generate the geometry on GiD, it is necessary to create a parametric surface. This surface is characterized by the following parametric equations

$$\begin{cases} x(u,v) = u \\ y(u,v) = v \\ z(u,v) = uv \end{cases} \tag{1}$$

Being the origin of coordinates at the central point. In this case $u, v \in [-5, 5]$. The parametric surface is defined by 100 points. Once the surface is generated, the `Mat_fem_Shells` GiD interface is used to assign the thickness and the material properties of the concrete to the solid, which are already available and are:

$$\begin{cases} \texttt{Young} = 2.1 \times 10^{11} MPa \\ \texttt{Poison} = 0.2 \\ \texttt{Self\_weight} = 78000 N/m^3 \\ \texttt{Thickness} = 0.1 m \end{cases} \tag{2}$$

After activating the self-weight as the only load, the structured mesh is computed as asked and the result is 162 triangular elements and 100 nodes, which can be seen in Fig. 1.



(a) Material properties assigned to the meshed surface.

(b) Meshed hyerbolic-paraboloid surface.

Figure 1: Mesh and material properties.

The boundary conditions result from constraining all degrees of freedom for the nodes on the boundary, thus obtaining a completely clamped shell on the boundary, as can be seen in Fig. 2.

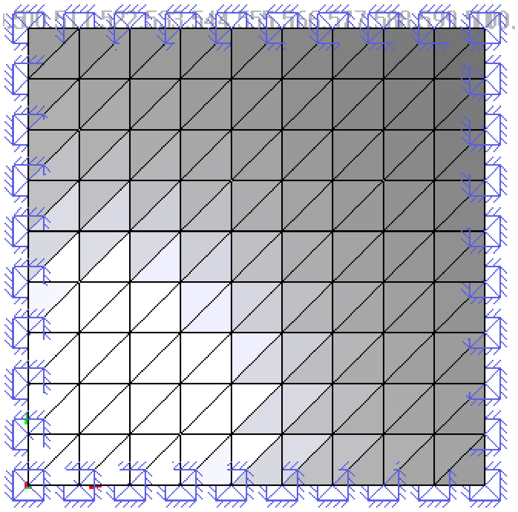
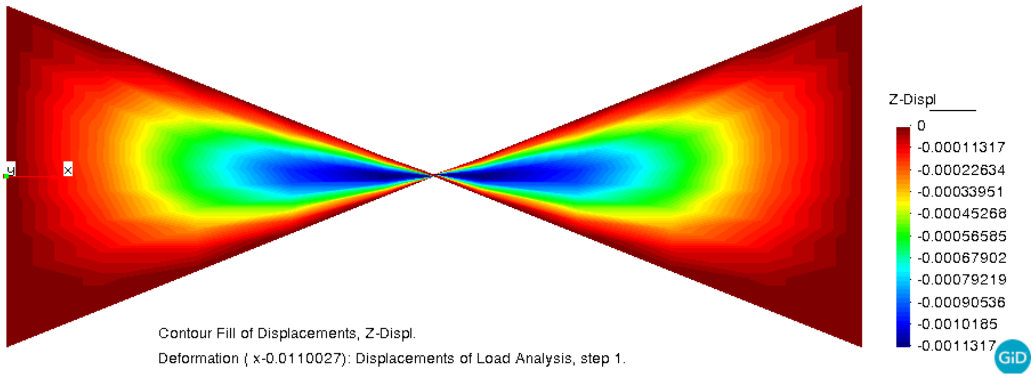


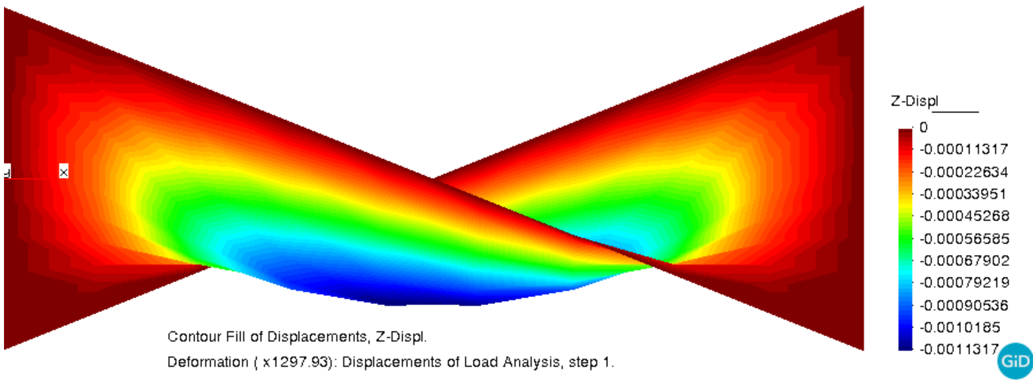
Figure 2: Boundary conditions.

The resulting file after calculation is used as input file to the Matlab's code `Lamina_T_RM`. The Matlab code is executed and the resulting `.flavia.res` file is used as input file for GiD's post-processor. As it is only asked to analyze the stresses, first the results are going to be assessed by analyzing the deformed geometry and the resulting displacements.

In like manner, the comparison between the original and deformed shapes (with the corresponding indicated magnification factor) are shown in Fig. 3.



(a) z-displacements on non-deformed shape.

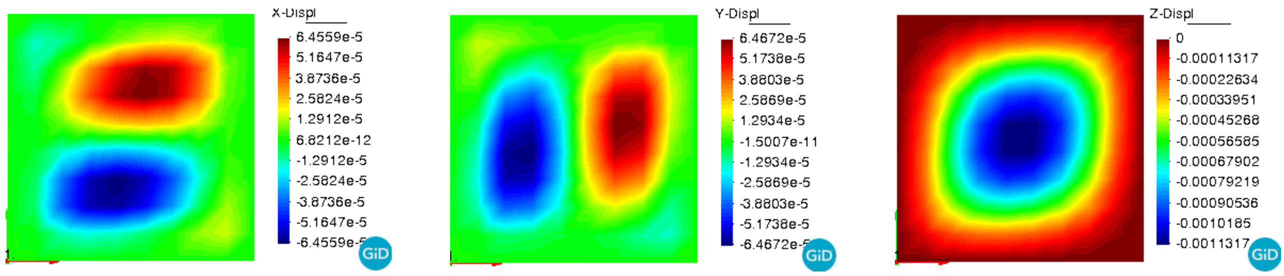


(b) z-displacements on deformed shape (× 1300).

Figure 3: z-displacements and and deformed shape.

Being the maximum displacement of the order of 1mm at the centre of the shell, it is clear that the displacements in the z direction are much smaller than that of a plate of the same dimensions, as the curvature provides the stiffness. Then, the displacements are smaller at high points, which was also to be expected. However, it is to be noticed that typically the dead load of such structures is small, and higher forces are to be expected from the wind or snow loads, which are also unbalanced. Then, the movement restraining would also to be expected at the two low points, not along all the perimeter.

Then the displacements seen from the Original XY plane are shown in Fig. 4. As could expected from a double curved surface will completely clamped supports, the displacement for the x and y coordinates are simply the same in magnitude but rotated ninety degrees. Being the displacements in the logical range of values, the assessment is done and the stresses can be evaluated.

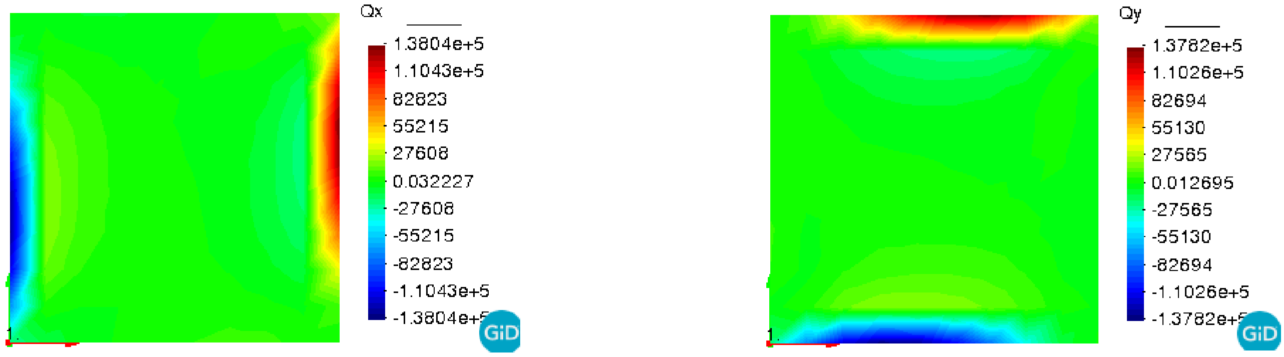


(a) x-displacement. (b) y-displacement. (c) z-displacement

Figure 4: Displacements on the membrane seen from the original XY plane.

Eventually, the shear stresses and membrane stresses are shown in Fig. 5 and 6, respectively. It is extracted that there is a shearing force along the length of the perimeter, so that the stresses in the sheathing result in boundary shear along the perimeter. The stresses are equal in magnitude and have opposite signs.

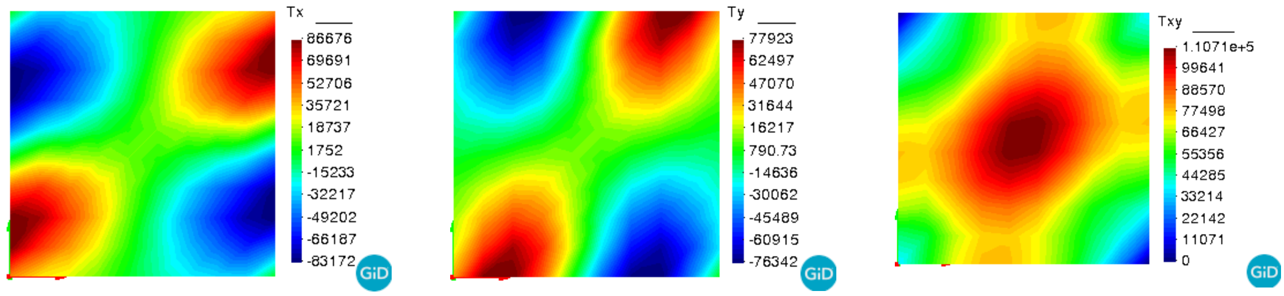
As for the membrane stresses, the component of the compressing stress normal to the perimeter member exerts an outward thrust on the perimeter and the component of the tensile stress normal to the perimeter exerts an inward pull. These components, being equal and opposite in direction, as can be seen from Fig. 6, will cancel each other, with the result that the perimeter members are subjected only to axial compression forces [1]. As could be expected, T_{xy} presents a radial increase towards the center whereas the others accumulate on both perimeter sides of the convex and concave parabola.



(a) Shear stress Q_x.

(b) Shear stress Q_y.

Figure 5: Shear stresses.



(a) Membrane stress T_x.

(b) Membrane stress T_y.

(c) Membrane stress T_{xy}.

Figure 6: Membrane stresses

References

[1] *Hyperbolic Paraboloid Shells*, Douglas Fir Use Book, Structural Data and Design Tables.