Universitat Politècnica de Catalunya
Master of Science in Computational Mechanics
Computational Structural Mechanics and Dynamics
CSMD
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Assignment 9 - Shells of Revolution FEM modelling

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Question A

As the Shell of revolution or any other revolution formulation an axi-symmetry is assumed in loads and supports, the only way to still use this formulation (In a modified form though)is to address the problem by modeling the complete structural configuration using 3-Dimensional model and applying the load at the appropriate location in the configuration.

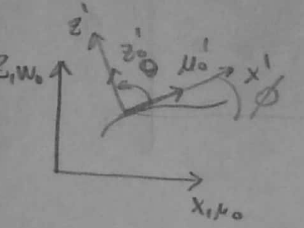
In a practical way, the angle θ around the symmetric axis is discretized by linear finite elements. Thus the formulation now has new dimension, like a FEM solution in time for example where t is the new dimension. So, instead of applying 2π the PVW integral, the angle θ is integrated element by element in the tangential direction from zero to the circumferential length L.

In order to easy the application for discontinuous loads in the circumferential domain, some authors [1] proposed the use of Fourier Series. In this approach, for consistency, the displacements an rotations are also discretized using a Fourier series.

References

[1] Jaya Lekshmi R, Sanju Mary Sobichen, M.K Sundaresan, and R Marimuthu. Axisymmetric solid with non-axisymmetric load using matlab. *International Journal of Scientific and Engineering Research*, 7(10), 2016.

Question B



b) for a straight element with $R \to \infty$

$$
\begin{cases}
u' = -z\,\theta_s = -z\dfrac{\partial w_0'}{\partial s} \\
u_0 = u_0 \\
w_0 = w_0 \\
\theta_s = \dfrac{\partial w_0}{\partial s}
\end{cases}
$$

(u and u_0 are independent)

• such that no shear strains are present $\gamma_{xz} = 0$

the generalized strains can be written as:

$$
\underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{x'} \\ \varepsilon_{\varphi} \\ \cancel{\gamma_{xz}} = 0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0'}{\partial s} + \dfrac{\partial u'}{\partial s} \\ \dfrac{u_0' \cos\phi}{r} + \dfrac{u' \cos\phi}{r} - \dfrac{w_0' \sin\phi}{r} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0'}{\partial s} - z\dfrac{\partial^2 w_0}{\partial s^2} \\ \dfrac{u_0' \cos\phi}{r} - z\dfrac{\partial w_0}{\partial s}\dfrac{\cos\phi}{r} - w_0\dfrac{\sin\phi}{r} \end{Bmatrix} \qquad (1)
$$

assuming the discretization of u and w such that:

$$
\begin{cases}
u_0 = \sum_{i=1}^{2} \hat{N}_i\, u_i \\
w_0 = \sum_{i=1}^{2} N_i\, w_i + \bar{N}_i \left(\dfrac{dw}{ds}\right)_i
\end{cases} \qquad (2)
$$

· N_i and $\bar{N}_i$ are C_1 continuity shape functions

Equation (1) can be written as:

$$
\underline{\varepsilon} = \underline{\varepsilon}_m + \underline{\varepsilon}_b = \begin{Bmatrix} \dfrac{\partial u_0'}{\partial s} \\ \dfrac{u_0' \cos\phi}{r} - \dfrac{w_0 \sin\phi}{r} \end{Bmatrix} + \begin{Bmatrix} -z \cdot \dfrac{\partial^2 w_0'}{\partial s^2} \\ -z\dfrac{\partial w_0}{\partial s}\dfrac{\cos\phi}{r} \end{Bmatrix}
$$

• b → bending
• m → membrane

$$
\hat{\underline{\varepsilon}} = \begin{Bmatrix} \hat{\underline{\varepsilon}}_m \\ \hat{\underline{\varepsilon}}_b \end{Bmatrix}, \quad \text{where} \quad \underline{\varepsilon}_b = z\,\hat{\underline{\varepsilon}}_b
$$

using the interpolation formulation of (2):

$$
\hat{\underline{\varepsilon}}_{m_i} = \begin{bmatrix} \dfrac{\partial \hat{N}_i}{\partial s} & 0 & 0 \\ \dfrac{\hat{N}_i \cos\phi}{r} & -\dfrac{N_i \sin\phi}{r} & -\dfrac{\bar{N}_i \sin\phi}{r} \end{bmatrix} \begin{Bmatrix} u_{0i}' \\ w_{0i}' \\ \left(\dfrac{dw_0'}{ds}\right)_i \end{Bmatrix} \qquad (3)
$$

$$\underline{\varepsilon}_b = \begin{bmatrix} 0 & -\frac{\partial^2 \hat{N}_i}{\partial s^2} & -\frac{\partial^2 \bar{N}_i}{\partial s^2} \\ 0 & -\frac{\partial \hat{N}_i}{\partial s}\frac{\cos\phi}{r} & -\frac{\partial \bar{N}_i}{\partial s}\frac{\cos\phi}{r} \end{bmatrix} \begin{Bmatrix} u_{0i}' \\ w_{0i}' \\ \left(\frac{\partial w_0'}{\partial s}\right)_i \end{Bmatrix}$$

thus $\underline{B}_i$ can be written as:

$$\underline{\underline{B}}_i = \begin{bmatrix} \frac{\partial \hat{N}_i}{\partial s} & 0 & 0 \\ \frac{\hat{N}_i \cos\phi}{r} & -\frac{N_i \sin\phi}{r} & -\frac{\bar{N}_i \sin\phi}{r} \\ 0 & -\frac{\partial^2 N_i}{\partial s^2} & -\frac{\partial^2 \bar{N}_i}{\partial s^2} \\ 0 & -\frac{\partial N_i}{\partial s}\frac{\cos\phi}{r} & -\frac{\partial \bar{N}_i}{\partial s}\frac{\cos\phi}{r} \end{bmatrix}$$

- as N_i and $\bar{N}_i$ are third order functions of the coordinates, an exact integration would require two Gauss points $(-1/\sqrt{3}, 1/\sqrt{3})$
- the resultant B matrix is a non-symmetric shaped one.