Assignment 9 – Axisymmetric shells and arcs

Computational Structural Mechanics and Dynamics

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1. Describe how a non-symmetric load can be applied in this formulation.

If a non-symmetric load should be applied in the formulation for axisymmetric shell structures, do the loads and displacements need to be expanded in Fourier series in the circumferential direction. This means we move into 3D and we will have to use another formulation. The finite strip and finite prism method, where the governing unknowns are represented by Fourier series, is a way of solving axisymmetric shell with arbitrary loading. This means the displacement will be presented in the following way

$$\sum_{l=1}^{m}\left(w^l(x,z)\sin\frac{l\pi}{b}y+\bar{w}^l(x,z)\cos\frac{l\pi}{b}y\right)$$

Where w^l and $\bar{w}^l$ are the l-th modal displacement amplitudes and m is the number of harmonic terms chosen for the analysis. The problem is then solved by adding up a number of standard finite element solutions for each harmonic term of the Fourier expansion

2. Using thin beam formulation, describe the shape of the B-matrix and comment on the integration rule.

When using the thin beam formulation, we have to follow Kirchhoff's assumptions, which differ from the Reissner-Minldin theory, by considering a small thickness that does not give a rotation of the normal of the plate. I.e. the Kirchhoff orthogonality condition is equivalent to neglecting the effect of the transverse shear deformation. This gives a B-matrix with terms regarding the membrane strain and bending strain.

$$B'_{m_i}=\begin{bmatrix}\frac{\partial N_i^u}{\partial s} & 0 & 0\\ \frac{N_i^w cos\varphi}{r} & -\frac{N_i^w sin\varphi}{r} & -\frac{\bar{N}_i^w sin\varphi}{r}\end{bmatrix}$$

$$B'_{b_i}=\begin{bmatrix}0 & \frac{\partial^2 N_i^w}{\partial s^2} & \frac{\partial^2 \bar{N}_i^w}{\partial^2 s}\\ 0 & \frac{cos\varphi}{r}\frac{\partial N_i^w}{\partial s} & \frac{cos\varphi}{r}\frac{\partial \bar{N}_i^w}{\partial s}\end{bmatrix}$$

Where the indexes m and b means the strain matrix for membrane and bending. The difference from the Reissner-Mindlin theory is therefore just one term less, which is the shear strain matrix.

For integration rule it is satisfactory to use a two-point quadrature for computing the integrals, but a more accurate result can be obtained using more points. However, good results are also obtained with the simplest reduced one-point quadrature. In cases where the bending and transverse shear are negligible, should it be used two- and three-point quadrature for 2-noded and 3-noded elements, respectively.