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Computational Structural Mechanics and Dynamics
Assignment 9 - Axisymmetric Shells

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1 Problem A

Problem statement: *Describe in extension how a non-symmetric load can be applied on this formulation.*

In the case where non-symmetric loads are applied to an axisymmetric structure, tangential displacements will become non-zero and must be accounted for in the shell formulation. According to Zienkiewicz and Taylor (see Reference 1), the problem of an axisymmetric solid subjected to non-symmetric loads may be solved by expressing displacements, as well as force components, as **Fourier series expansions**, splitting them into symmetric and antisymmetric components.

For the formulation of axisymmetric thin shells (Euler-Bernoulli formulation), the definition of strains must also be modified to take into account displacements and force components in all three directions. The strain vector then becomes:

$$\overline{\varepsilon} = \begin{bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \gamma_{s\theta} \\ \chi_s \\ \chi_\theta \\ \chi_{s\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \overline{u}}{\partial s} \\ \frac{1}{r}\frac{\partial \overline{v}}{\partial \theta} + (\overline{u}cos\phi - \frac{1}{r}\overline{w}sin\phi) \\ \frac{1}{r}\frac{\partial \overline{u}}{\partial \theta} + \frac{\partial \overline{v}}{\partial s} - \frac{1}{r}\overline{v}cos\phi \\ -\frac{\partial^2 \overline{w}}{\partial s^2} \\ -\frac{1}{r^2}\frac{\partial^2 \overline{w}}{\partial \theta^2} - \frac{1}{r}\frac{\partial \overline{w}}{\partial s}cos\phi + \frac{1}{r}\frac{\partial \overline{v}}{\partial \theta}sin\phi \\ 2(-\frac{1}{r}\frac{\partial^2 \overline{w}}{\partial s \partial \theta} + \frac{1}{r^2}\frac{\partial \overline{w}}{\partial \theta} + \frac{1}{r}\frac{\partial \overline{v}}{\partial s} - \frac{1}{r^2}\overline{v}sin\phi cos\phi) \end{bmatrix} \tag{1}$$

Therefore, three membrane and three bending effects are now present in the stress vector:

$$\sigma = \begin{bmatrix} N_s & N_\theta & N_{s\theta} & M_s & M_\theta & M_{s\theta} \end{bmatrix}' \tag{2}$$

2 Problem B

Problem statement: *Using thin beams formulation, describe the shape of the $B^{(e)}$ matrix and comment the integration rule.*

For the Kirchhoff formulation of axisymmetric shells, shear effects are neglected and the **B** matrix for the i^{th} element becomes:

$$B_i = \begin{bmatrix} B_{m_i} \\ B_{b_i} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_i^u}{\partial s} & 0 & 0 \\ \frac{N_i^u cos\phi}{r} & -\frac{N_i^w sin\phi}{r} & -\frac{\overline{N}_i^w sin\phi}{r} \\ 0 & \frac{\partial^2 N_i^w}{\partial s^2} & \frac{\partial^2 \overline{N}_i^w}{\partial s^2} \\ 0 & \frac{cos\phi}{r}\frac{\partial N_i^w}{\partial s} & \frac{cos\phi}{r}\frac{\partial \overline{N}_i^w}{\partial s} \end{bmatrix} \tag{3}$$

Regarding the integration rule, it may be observed that several terms of matrix **B** contain divisions by r. This could mean that an integration method for which it is necessary to evaluate the integrals at the nodes would present problems at nodes with $r = 0$.

The inconvenient is however solved by using a numerical integration scheme such as Gauss quadratures, where the function need not be evaluated at the nodes. According to Oñate (see reference 2), results obtained with a simple reduced one-point quadrature provide sufficient precision.

3 References

[**1**] Zienkiewicz, O.C.; Taylor, R.L.The Finite Element Method Volume 2: Solid Mechanics, Fifth Edition. Butterworth Heinemann, 2000

[**2**] Oñate, E. Structural Analysis with the Finite Element Method, Linear Statics Vol. 2: Beams, Plates and Shells, First Edition. Springer, 2013