Universitat Politècnica De Catalunya, Barcelona

MSc. Computational Mechanics Erasmus Mundus

Assignment 9: Rev. Shells

Computational Structural Mechanics & Dynamics

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Problem a):

Describe in extension how can be applied a non-symmetric load on this formulation.

Solution:

The axisymmetric analysis of a structure is usually based on the assumption that the load applied is also symmetric with respect to the axis of symmetry. If a non-symmetric load is to be applied to an axisymmetric structure, we tend to use the entire three-dimensional model and apply the load at the specific positions of the geometry. Although, this method would be computationally expensive compared to an equivalent axisymmetric model.

In order to use a two-dimensional structure configuration, we could express the non-symmetric load as a series of harmonic functions or Fourier series i.e. as a loading pattern over the revolution angle i.e. 0 to 360 degrees.

For example, a load F can be given as,

$$F(\theta) = A_0 + A_1\ cos\theta + B_1\ sin\theta + A_2\ cos2\theta + B_2\ sin2\theta + A_3\ cos3\theta + B_3\ sin3\theta + ...$$

Obtaining a 2-D solution for each Fourier component would help us in generating a general solution for the desired angle (within 0 - 360 degrees) over the geometry.

Problem b):

Using thin beams formulation, describe the shape of the $B^{(e)}$ matrix and comment the integration rule.

Solution:

As Timoshenko beam theory takes into account the effect of shear strains in the deformation energy, we have the strains as,

$$\epsilon_x = \frac{du}{dx} = -z\frac{d\theta}{dx}$$

$$\gamma_{xz} = \frac{dw}{dx} + \frac{du}{dz} = \frac{dw}{dx} - \theta = -\phi$$

where the angle ϕ is the measure of shear strains.

For a linear interpolation of w and θ, we choose,

$$w(\xi) = N_1(\xi)w_1 + N_2(\xi)w_2$$

$$\theta(\xi) = N_1(\xi)\theta_1 + N_2(\xi)\theta_2$$

and therefore curvature and shear strain are given as,

$$\chi = \frac{d\theta}{dx} = \frac{d\xi}{dx}\frac{d\theta}{d\xi} = \frac{d\xi}{dx}(\frac{dN_1}{d\xi}w_1 + \frac{dN_2}{d\xi}w_2) - (N_1\theta_1 + N_2\theta_2)$$

$$\gamma_{xz} = \frac{dw}{dx} - \theta = \frac{d\xi}{dx}(\frac{dN_1}{d\xi}w_1 + \frac{dN_2}{d\xi}w_2) - (N_1\theta_1 + N_2\theta_2)$$

We know,

$$\chi = \boldsymbol{B}_b\boldsymbol{a}^{(e)} \quad \text{and} \quad \gamma_{xz} = \boldsymbol{B}_s\boldsymbol{a}^{(e)}$$

Hence, with

$$\boldsymbol{a}^{(e)} = \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix}^T$$

we can write the B-matrices for curvature and shear strain as,

$$\boldsymbol{B}_b = \begin{bmatrix} 0 & \frac{2}{l^{(e)}}\frac{dN_1}{d\xi} & 0 & \frac{2}{l^{(e)}}\frac{dN_2}{d\xi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{l^{(e)}} & 0 & \frac{1}{l^{(e)}} \end{bmatrix}$$

$$\boldsymbol{B}_s = \begin{bmatrix} \frac{2}{l^{(e)}}\frac{dN_1}{d\xi} & -N_1 & \frac{2}{l^{(e)}}\frac{dN_2}{d\xi} & -N_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{l^{(e)}} & \frac{-(1-\xi}{2} & \frac{1}{l^{(e)}} & \frac{-(1+\xi}{2} \end{bmatrix}$$

Now for tronco-conical plate elements and using linear shape functions as Timoshenko beam theory (since we do not need C^1 continuity) we have,

$$\boldsymbol{u'} = \sum_{i=1}^{n} \boldsymbol{N}_i \boldsymbol{a'}_i^{(e)}$$

where,

$$\boldsymbol{N}_i = \begin{bmatrix} N_i(\xi) & 0 & 0 \\ 0 & N_i(\xi) & 0 \\ 0 & 0 & N_i(\xi) \end{bmatrix} \quad \text{and} \quad \boldsymbol{a'}_i^{(e)} = \begin{bmatrix} u'_{0i} \\ w'_{0i} \\ \theta_i \end{bmatrix}$$

The local generalised strain vector is,

$$\epsilon' = \begin{bmatrix} \epsilon'_m \\ \epsilon'_f \\ \epsilon'_0 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u'_0}{\partial s} \\ \dfrac{u'_0 cos\phi - w'_0 sin\phi}{x} \\ -\dfrac{\partial \theta}{\partial s} \\ -\dfrac{\theta cos\phi}{x} \\ \dfrac{\partial w'_0}{\partial s} - \theta \end{bmatrix}$$

It is interesting to note here that the above deformation vector consists of the matrices of the bending and shear terms explained above i.e. the axial strain ($\frac{\partial u'_0}{\partial s}$), circumferential bending ($-\frac{\theta cos\phi}{x}, \frac{u_0}{x}$) and the Timoshenko beam theory ($\frac{\partial \theta}{\partial s}, \frac{\partial u'_0}{\partial s - \theta}$).

Therefore, the membrane, bending and shear B-matrices are written as,

$$\boldsymbol{B'}_{mi} = \begin{bmatrix} \dfrac{\partial N_i}{\partial s} & 0 & 0 \\ \dfrac{N_i cos\phi}{x} & -\dfrac{N_i sin\phi}{x} & 0 \end{bmatrix}$$

$$\boldsymbol{B'}_{bi} = \begin{bmatrix} 0 & 0 & \dfrac{\partial N_i}{\partial s} \\ 0 & 0 & -\dfrac{N_i cos\phi}{x} \end{bmatrix}$$

$$\boldsymbol{B'}_{si} = \begin{bmatrix} 0 & \dfrac{\partial N_i}{\partial s} & -N_i \end{bmatrix}$$

where, s is the length of the element, x is the position at which we evaluate this matrices.

We realise that it is impossible to evaluate these terms at the axis of symmetry (where $x = 0$). Also, if this position is very large i.e. tending to infinity, the terms $1/x$ tend to zero. The global B-matrix after transformation is given as,

$$\boldsymbol{B}_i = \boldsymbol{B'}_i \boldsymbol{L}^T = \begin{bmatrix} \boldsymbol{B}_{mi} \\ \boldsymbol{B}_{bi} \\ \boldsymbol{B}_{si} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial N_i}{\partial s} cos\phi & \dfrac{\partial N_i}{\partial s} sin\phi & 0 \\ \dfrac{N_i}{x} & 0 & 0 \\ 0 & 0 & -\dfrac{\partial N_i}{\partial s} \\ 0 & 0 & -\dfrac{N_i cos\phi}{x} \\ -\dfrac{\partial N_i}{\partial s} sin\phi & \dfrac{\partial N_i}{\partial s} cos\phi & -N_i \end{bmatrix}$$

The above three matrices, $\boldsymbol{B}_m, \boldsymbol{B}_b$ and $\boldsymbol{B}_s$ are implemented in the Finite Element formulation of size $2 \times 6, 2 \times 6$ and 1×6, respectively for a linear element. Looking at the shear B-matrix, we notice that our choice of linear shape functions makes the global stiffness matrix a quadratic one, which therefore requires at least two gauss points along the element for the numerical integration. In general, reduced integration is used to avoid the shear locking effect by using only one integration gauss point, gauss point 1 = gauss point 2 = 0.5.