UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Universitat Politècnica de Catalunya

Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports

MASTER EN INGENIERÍA ESTRUCTURAL Y DE LA CONSTRUCCIÓN

Course:

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 6

On "Beams"

By

Sierra, Pablo Leonel

Assignment 6:

a) Program in Mat Lab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix.

$$\mathbf{K}_b^{(e)} = \left(\frac{EI}{l}\right)^{(e)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \text{(The point interpolation is exact for } \mathbf{K}_b^{(e)}\text{)}$$

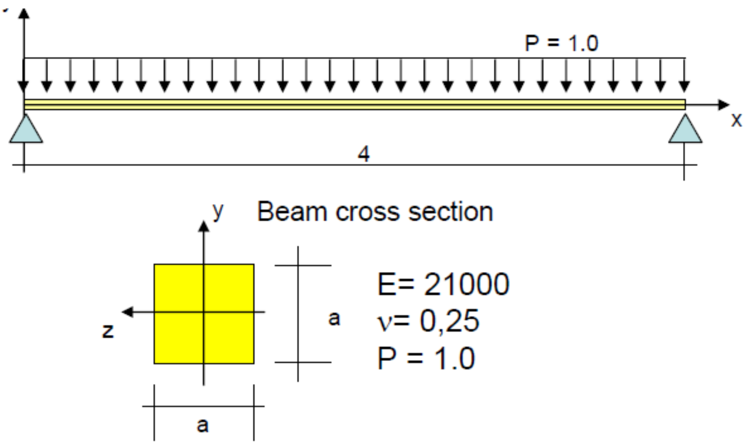
$$\mathbf{K}_s^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \cdots & 1 & -\frac{l^{(e)}}{2} \\ \text{Simetr.} & & \cdots & \frac{(l^{(e)})^2}{4} \end{bmatrix} \text{(Reduced integration)}$$

Hint: For stress evaluation make gaus1 = gaus2 = 0.0

b) Solve the following problem with a 64 element mesh with the:

- 2 nodes Euler Bernulli element
- 2 nodes Timoshenko Full Integrate element
- 2 nodes Timoshenko Reduce Integration element.

Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.



a = 0,001

a = 0,005

a = 0,010

a = 0,020

a = 0,050

a = 0,100

a = 0,200

a = 0,400

---

a) Using the Mat_Fem program for MATLAB, its used as base the Timoshenko beam program. At first, is changed the $K_s^{(e)}$

```
K_s = [   1    ,    len/2 ,    -1    ,    len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ;
         -1    ,   -len/2 ,     1    ,   -len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ];
```

The Stress function used in the element is also modified to reduce the gauss points used. For that, instead of use 2 point in $\xi = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$, are used $\xi = 0$.

```
% Two gauss point for stress evaluation
    % gaus1 =-1/sqrt(3);
    % gaus2 = 1/sqrt(3);
    gaus1 = 0;
    gaus2 = 0;
    bmat_b = [ 0, -1/len, 0, 1/len];
    bmat_s1=[-1/len,-(1-gaus1)/2, 1/len,-(1+gaus1)/2];
    bmat_s2=[-1/len,-(1-gaus2)/2, 1/len,-(1+gaus2)/2];
    Str1_g0 = D_matb*(bmat_b *transpose(u_elem));
    Str2_g0 = D_mats*(bmat_s1*transpose(u_elem));
    Str3_g0 = D_mats*(bmat_s2*transpose(u_elem));

    Strnod(lnods(1),1) = Strnod(lnods(1),1) + Str1_g0;
    Strnod(lnods(2),1) = Strnod(lnods(2),1) + Str1_g0;
    Strnod(lnods(1),2) = Strnod(lnods(1),2) + Str2_g0/2
+ Str3_g0/2;
    Strnod(lnods(2),2) = Strnod(lnods(2),2) + Str2_g0/2
+ Str3_g0/2;
    Strnod(lnods(1),3) = Strnod(lnods(1),3) + 1;
    Strnod(lnods(2),3) = Strnod(lnods(2),3) + 1;
```

b) At first, the geometrical properties of the beam are calculated for the different relationship given

a/l	a	A	I
0.001	0.004	1.600000E-05	2.133333E-11
0.005	0.02	4.000000E-04	1.333333E-08
0.010	0.04	1.600000E-03	2.133333E-07
0.020	0.08	6.400000E-03	3.413333E-06
0.050	0.2	4.000000E-02	1.333333E-04
0.100	0.4	1.600000E-01	2.133333E-03
0.200	0.8	6.400000E-01	3.413333E-02
0.400	1.6	2.560000E+00	5.461333E-01

The input data for $\frac{a}{l} = 0.01$ si shown next

```
%=============================================================
===========
% MAT-fem_Beams 1.0  - MAT-fem is a learning tool for
undestanding
%                       the Finite Element Method with MATLAB
and GiD
%=============================================================
===========
% PROBLEM TITLE = Untitled
%
%  Material Properties
%
  young   =   2.100000000e+04 ;
  poiss   =  2.500000000e-01 ;
  denss   =   0.000000000e+00 ;
  area    =   1.600000000e-03 ;
  inertia=   2.133333333e-07 ;
%
% Coordinates
%
global coordinates
coordinates = [
     0.000000000E+00    ;
     6.250000000E-02    ;
     1.250000000E-01    ;
     1.875000000E-01    ;
     2.500000000E-01    ;
     3.125000000E-01    ;
     3.750000000E-01    ;
     4.375000000E-01    ;
     5.000000000E-01    ;
     5.625000000E-01    ;
     6.250000000E-01    ;
     6.875000000E-01    ;
     7.500000000E-01    ;
     8.125000000E-01    ;
     8.750000000E-01    ;
     9.375000000E-01    ;
     1.000000000E+00    ;
     1.062500000E+00    ;
     1.125000000E+00    ;
     1.187500000E+00    ;
     1.250000000E+00    ;
     1.312500000E+00    ;
     1.375000000E+00    ;
     1.437500000E+00    ;
     1.500000000E+00    ;
     1.562500000E+00    ;
     1.625000000E+00    ;
```

```
    1.687500000E+00    ;
    1.750000000E+00    ;
    1.812500000E+00    ;
    1.875000000E+00    ;
    1.937500000E+00    ;
    2.000000000E+00    ;
    2.062500000E+00    ;
    2.125000000E+00    ;
    2.187500000E+00    ;
    2.250000000E+00    ;
    2.312500000E+00    ;
    2.375000000E+00    ;
    2.437500000E+00    ;
    2.500000000E+00    ;
    2.562500000E+00    ;
    2.625000000E+00    ;
    2.687500000E+00    ;
    2.750000000E+00    ;
    2.812500000E+00    ;
    2.875000000E+00    ;
    2.937500000E+00    ;
    3.000000000E+00    ;
    3.062500000E+00    ;
    3.125000000E+00    ;
    3.187500000E+00    ;
    3.250000000E+00    ;
    3.312500000E+00    ;
    3.375000000E+00    ;
    3.437500000E+00    ;
    3.500000000E+00    ;
    3.562500000E+00    ;
    3.625000000E+00    ;
    3.687500000E+00    ;
    3.750000000E+00    ;
    3.812500000E+00    ;
    3.875000000E+00    ;
    3.937500000E+00    ;
    4.000000000E+00    ] ;
%
% Elements
%
global elements
elements = [
       1    ,      2    ;
       2    ,      3    ;
       3    ,      4    ;
       4    ,      5    ;
       5    ,      6    ;
       6    ,      7    ;
```

```
     7   ,      8   ;
     8   ,      9   ;
     9   ,     10   ;
    10   ,     11   ;
    11   ,     12   ;
    12   ,     13   ;
    13   ,     14   ;
    14   ,     15   ;
    15   ,     16   ;
    16   ,     17   ;
    17   ,     18   ;
    18   ,     19   ;
    19   ,     20   ;
    20   ,     21   ;
    21   ,     22   ;
    22   ,     23   ;
    23   ,     24   ;
    24   ,     25   ;
    25   ,     26   ;
    26   ,     27   ;
    27   ,     28   ;
    28   ,     29   ;
    29   ,     30   ;
    30   ,     31   ;
    31   ,     32   ;
    32   ,     33   ;
    33   ,     34   ;
    34   ,     35   ;
    35   ,     36   ;
    36   ,     37   ;
    37   ,     38   ;
    38   ,     39   ;
    39   ,     40   ;
    40   ,     41   ;
    41   ,     42   ;
    42   ,     43   ;
    43   ,     44   ;
    44   ,     45   ;
    45   ,     46   ;
    46   ,     47   ;
    47   ,     48   ;
    48   ,     49   ;
    49   ,     50   ;
    50   ,     51   ;
    51   ,     52   ;
    52   ,     53   ;
    53   ,     54   ;
    54   ,     55   ;
    55   ,     56   ;
```

```
     56    ,      57    ;
     57    ,      58    ;
     58    ,      59    ;
     59    ,      60    ;
     60    ,      61    ;
     61    ,      62    ;
     62    ,      63    ;
     63    ,      64    ;
     64    ,      65    ] ;
%
% Fixed Nodes
%
fixnodes = [
      1   , 1 ,    0.000000000e+00   ;
     65   , 1 ,    0.000000000e+00   ] ;
%
% Point loads
%
pointload = [ ] ;
%
% Side loadsss
%
uniload = -1*ones( 64 , 1);
```

Only changes the area and inertia for the others relations.

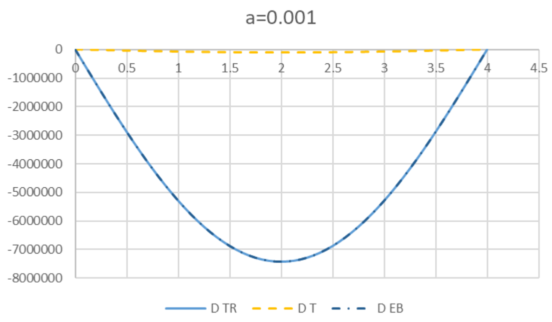
The maximum values obtained are shown in the next table

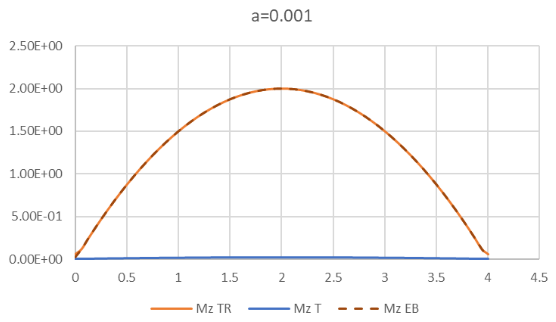
a/l	Maximum displacement			Maximum Mz			Maximum Q	
	TR	T	EB	Mz TR	Mz T	Mz EB	TR	T
0.001	-7.44E+06	-9.03E+04	-7.44E+06	2.00E+00	2.43E-02	2.00E+00	1.97E+00	1.97E+00
0.005	-1.19E+04	-2.80E+03	-1.19E+04	2.00E+00	4.70E-01	2.00E+00	1.97E+00	1.97E+00
0.010	-7.44E+02	-4.10E+02	-7.44E+02	2.00E+00	1.10E+00	2.00E+00	1.97E+00	1.97E+00
0.020	-4.65E+01	-3.87E+01	-4.65E+01	2.00E+00	1.66E+00	2.00E+00	1.97E+00	1.97E+00
0.050	-1.20E+00	-1.16E+00	-1.19E+00	2.00E+00	1.94E+00	2.00E+00	1.97E+00	1.97E+00
0.100	-7.62E-02	-7.56E-02	-7.44E-02	2.00E+00	1.98E+00	2.00E+00	1.97E+00	1.97E+00
0.200	-5.09E-03	-5.09E-03	-4.65E-03	2.00E+00	1.99E+00	2.00E+00	1.97E+00	1.97E+00
0.400	-4.02E-04	-4.02E-04	-2.91E-04	2.00E+00	2.00E+00	2.00E+00	1.97E+00	1.97E+00

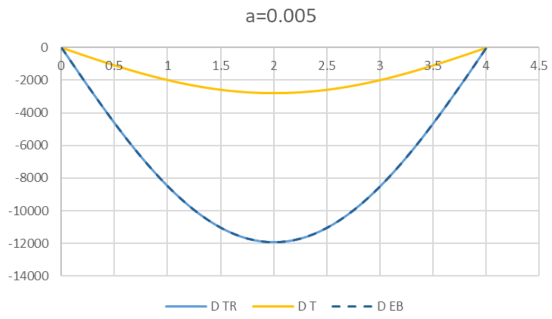
Where

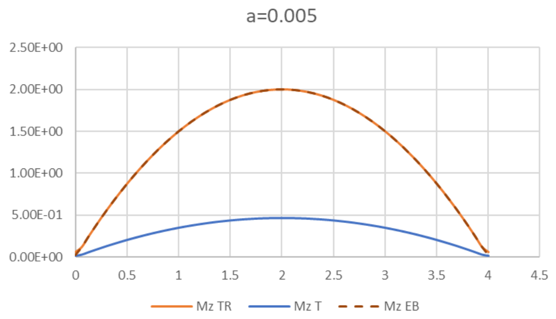
- TR: Timoshenko Reduced
- T: Timoshenko
- EB: Euler – Bernoulli

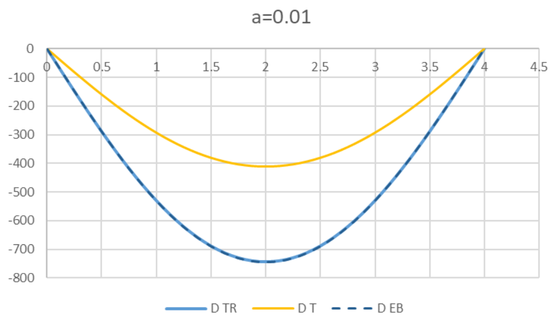
In the next figures is shown the distribution of the displacement (left) and the moment (right) in x, with the 3 theory's.

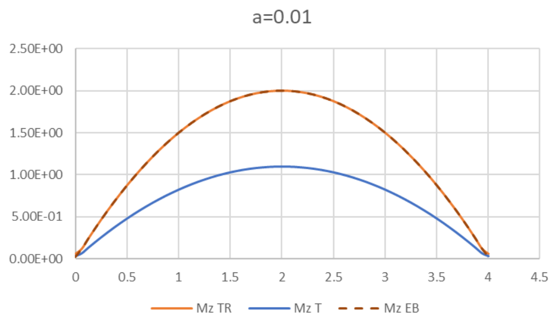

a=0.001
D TR
D T
D EB

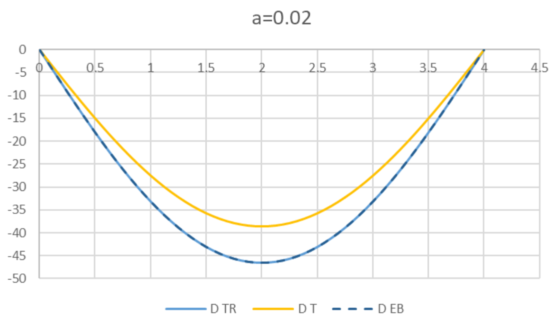

a=0.001
Mz TR
Mz T
Mz EB

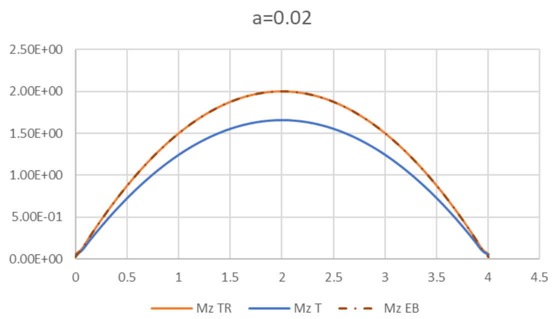

a=0.005
D TR
D T
D EB

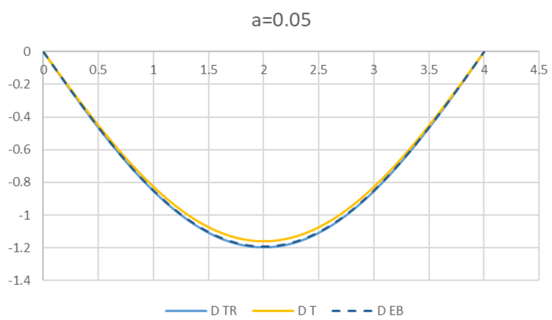

a=0.005
Mz TR
Mz T
Mz EB

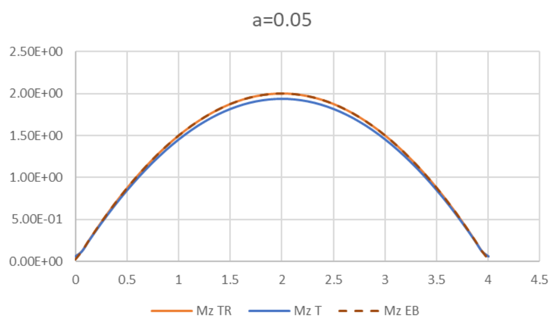

a=0.01
D TR
D T
D EB

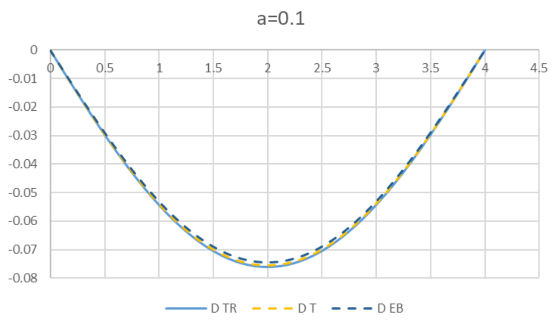

a=0.01
Mz TR
Mz T
Mz EB

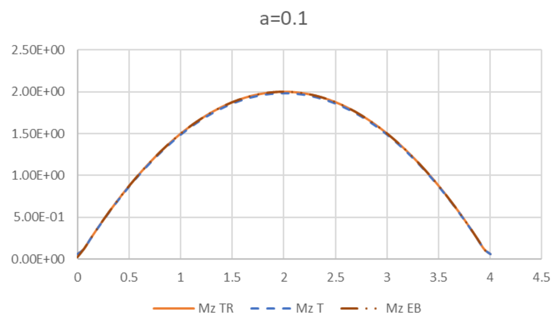

a=0.02
D TR
D T
D EB

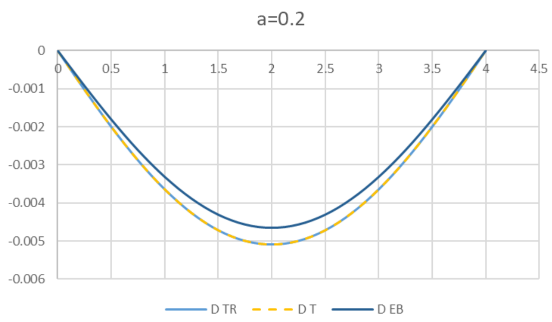

a=0.02
Mz TR
Mz T
Mz EB

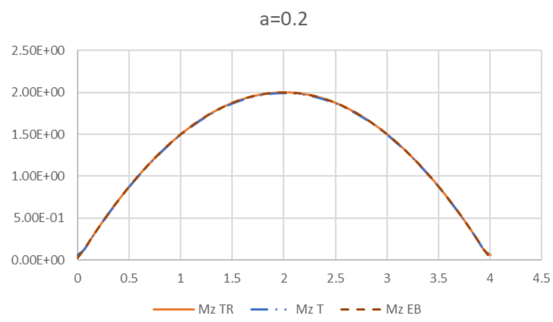

a=0.05
D TR
D T
D EB

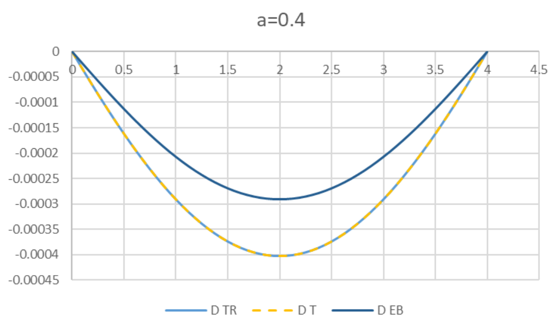

a=0.05
Mz TR
Mz T
Mz EB

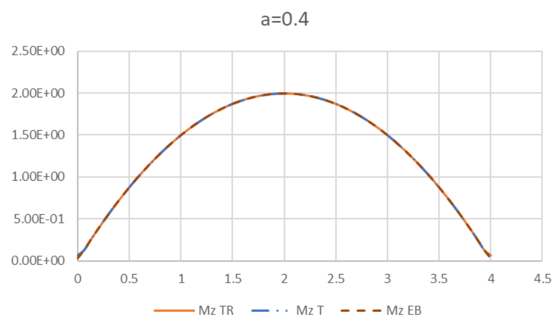












For the thinner beams, the shear locking effects on Timoshenko standard form makes the error in results are so important and the beam is stiffer than the reality. Instead of reach the Euler – Bernoulli theory, this form tends to zero displacement. In the reduced form, we can observe the theory approach the Euler – Bernoulli theory very well, avoiding this shear locking effect. For wider beams where the shear effects can't be avoided., the reduced model approach the standard Timoshenko form.

These previous comparisons are made considering the same discretization of the beam. The Timoshenko standard form converges extremely slowly, in terms of number of elements, to the exact solution than the EB and TR, the last one solves that problem.

In shear, both Timoshenko theory's delivered the same results.