Universitat Politècnica de Catalunya

Computational Solid Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Plate element analysis using Matlab

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Contents

1 The shear blocking effect on the Reissner Mindlin element

The purpose of this part of the assignment is to show that, for certain values of thickness for a plate, a divergence exist between the values of the maximum displacements produced on the plate for uniform loads applied on the element. For this purpose, a mesh is generated according to the specified parameters. The mesh consists of 25 nodes and 16 elements in total (5 ×5 mesh) so that the maximum displacement will clearly be on the central point of the plate. The mesh and numbering are shown in Fig. 1.

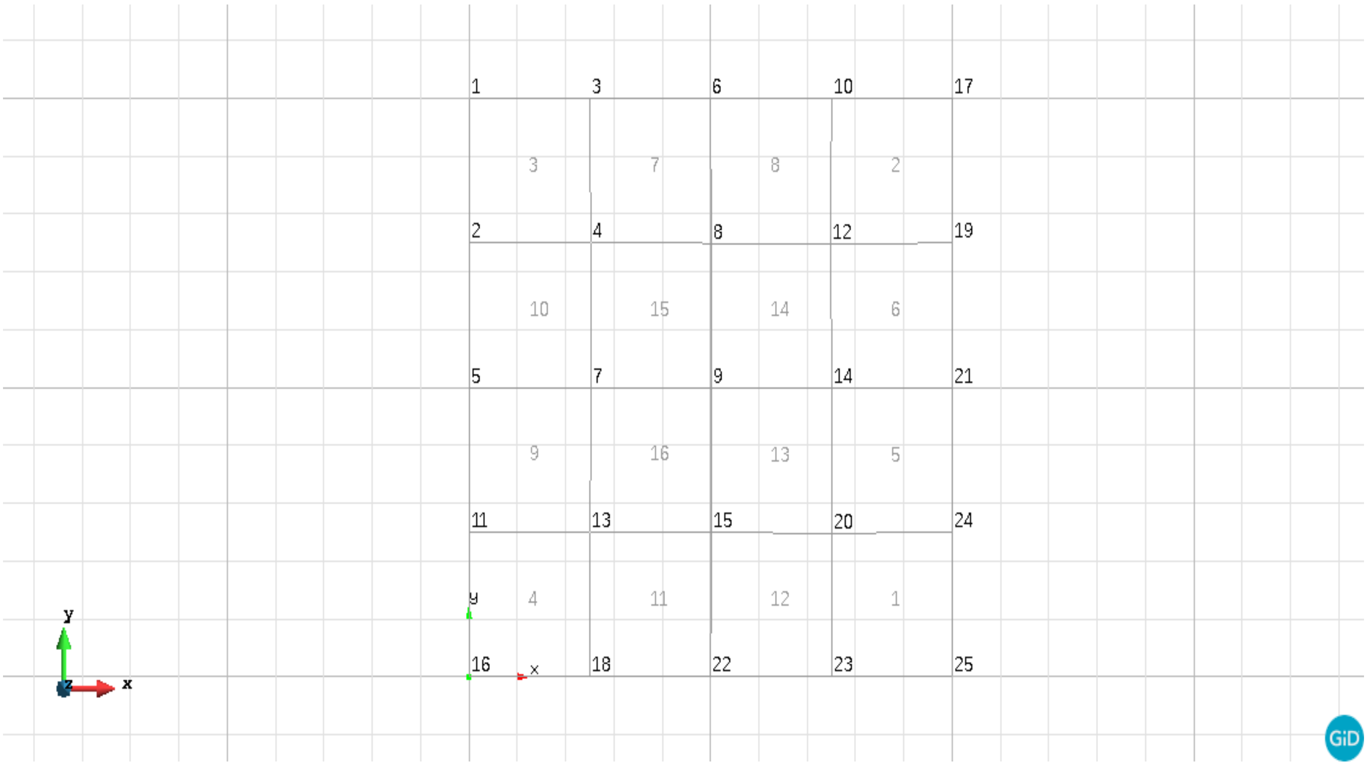

Figure 1: Mesh to be computed on.

These programs are designed so as to be used with an input file that collects the main variables of the problem so that the finite element code can return the computed variables. In doing so, the characteristics of this input file are summarized in the following box.

1 `input_file`

Brief description: Contains the values of the variables that the problem needs to be executed.
Input variables: Young's modulus (`young` = 10.92), Poisson's ratio (`poiss` = 0.3), density (`denss` = 0), coordinates of the nodes (`coordinates`), connectivity matrix (`elements`) and boundary conditions (`fixdesp`, `pointload`.

Output variables: [].

Now, it is very easy to generate the connectivity matrix and the coordinates vector for this case as it is a quadrilateral mesh with only 16 nodes and many problems have been developed so far to achieve this purpose. The connectivity matrix will be a matrix with as many columns as nodes per element(4) and 16 rows accounting for the elements. The point loads are computed so that each node receives the proportional load of the element. The nodes o the boundary have their displacement fixed. Furthermore, those parameters that depend on the thickness of the plate t, are defined inside each subroutine of calculation. Therefore, the two Matlab programs are defined as a function with the only input parameter being t, which is generated in an iterative loop that increases the value of t to the values proposed. In turn, each of these functions return the maximum value (absolute) of the stresses, moments and displacements. The results can be seen in Fig. 2.

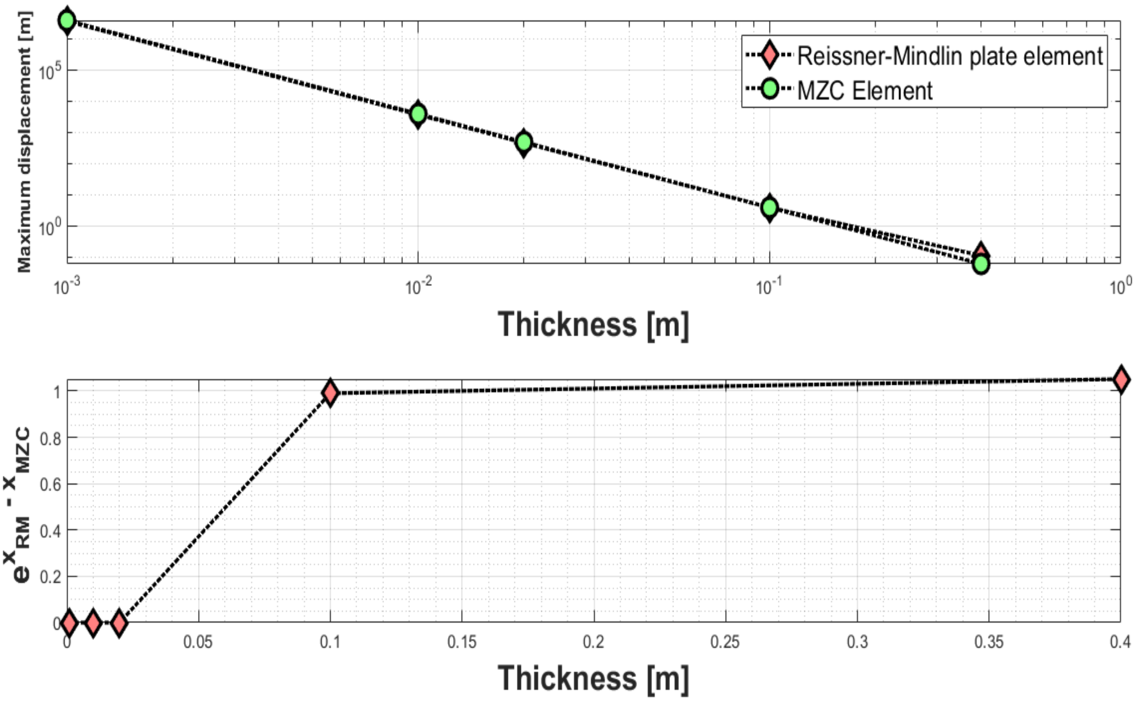

Figure 2: Evolution of the maximum values of displacement for different values of the t ratio.

As can be seen, it can be concluded that both the Reissner Mindlin and MZC elemental theory of plates are an extension of the already known Kirchhoff plate theory that takes into account shear deformations through-the-thickness of a plate. However, both values present (although not apparent at first sight) different values which do not present any physical sense. The latter is due as the low value for the Young's Modulus that we are given. The discrepancies are due to the fact that the RM element theory is intended for thick plates whose thickness is of the order of one tenth the planar dimensions while the MZC theory is applicable to thinner plates [1]. This is what can be seen in Fig. 2, specially on the second graph, where, to visualize the results more clearly, the normalized results of the difference for each value has been plotted. For values of thickness higher that $t = 0.1$, and given that the length of the domain is 1m, as the theory predicts, both values are close to each other due to the fact that in this case both theories are valid and applicable, and hence the convergence to value 1. For low values of the thichess, i.e. $t < 0.1$, the Reissner Mindlin elemental theory predicts a smaller displacement due to the blocking effect, which in turn comes from a prediction of an over-stiff element. For values higher that this, as commented, the value computed with both elements tends to convergence.

As a final conclusion, both discrepancies could also be predicted from the fact that the Reissner Mindlin elemental theory implies a linear displacement variation through the thickness, which is incompatible with MZC plate theory.

2 Patch test mesh for the MCZ element

The purpose of this part is to reproduce a test problem having an exact solution that can, in principle, be exactly reproduced by the numerical approximation. With this procedure, the quality of the MZC non-conforming finite element can be evaluated in the context of it being a component of a mesh. Considering a trivial domain of four quadrilateral elements and the patch node being the central node, the patch test is defined as assuming the following boundary conditions,

$$\begin{cases} u(x,0) = x \\ u(x+1) = x+1 \\ u(0,y) = y \\ u(1,y) = 1+y \end{cases} \tag{1}$$

with the following analytical solution $u(x, y) = x + y$ to be used to compare the computed solution. The exact values are obviously obtained when computing with the regular mesh, as can be seen in Fig. 3, provided that the length of the domain is still 1. For the visualization of the results, the program Paraview has been used.

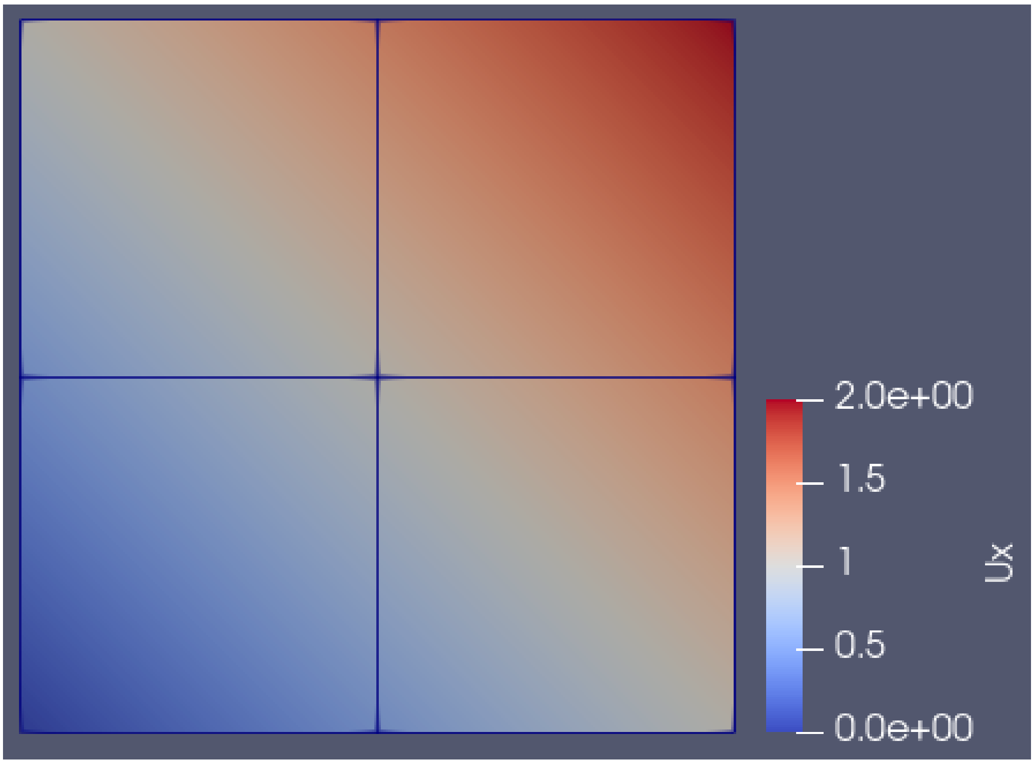

Figure 3: Computation for the regular mesh.

However, when considering a mesh with a distorted central node, it can be seen in Fig. 4 does not satisfy the patch test for the same conditions. This is due to the fact that the shape functions of the element do not hold with irregular meshes.

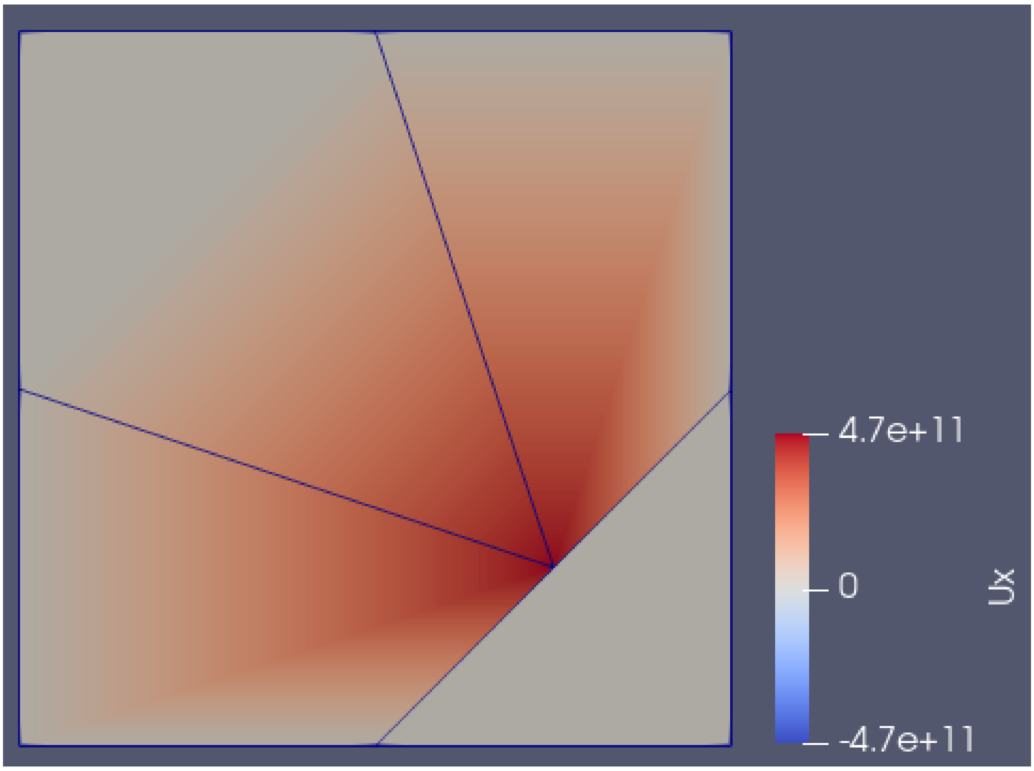

Figure 4: Computation for the irregular mesh.

References

[1] E. Reissner, 1945, *The effect of transverse shear deformation on the bending of elastic plates*, ASME Journal of Applied Mechanics, Vol. 12, pp. A68–77.