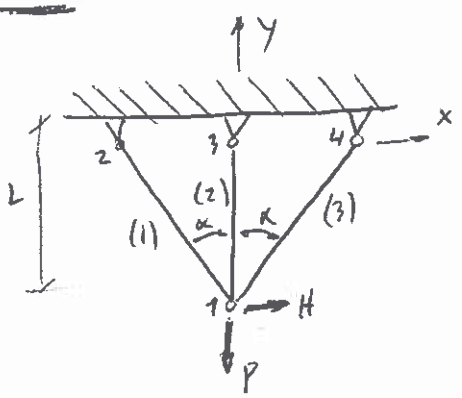
1.1



E and A same for all three bars

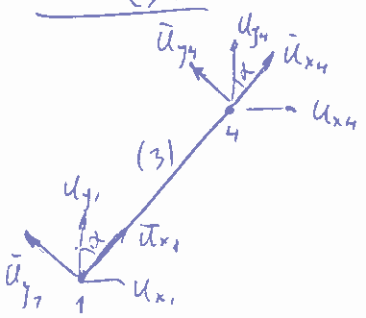
L, α, E, A, P, H – variables

fixed displacement nodes 2, 3, 4

$\alpha \neq 0$

$$L_1 = L_3 = \frac{L}{c}$$

Bar (3)



$$\bar{K}^3 = \frac{EAc}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad T^3 = \begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix}$$

$$\begin{cases} \bar{u}_{x1} = s\,u_{x1} + c\,u_{y1} \\ \bar{u}_{y1} = -c\,u_{x1} + s\,u_{y1} \end{cases} \qquad \begin{cases} \bar{u}_{x4} = s\,u_{x4} + c\,u_{y4} \\ \bar{u}_{y4} = -c\,u_{x4} + s\,u_{y4} \end{cases}$$

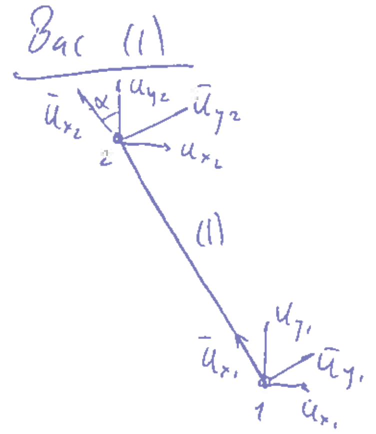
$$\bar{u}^3 = T^3 u^3 \Rightarrow \begin{bmatrix} \bar{u}_{x1} \\ \bar{u}_{y1} \\ \bar{u}_{x4} \\ \bar{u}_{y4} \end{bmatrix} = \begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

$$K^3 = T^{3T} \cdot \bar{K}^3 \cdot T^3 = \frac{EAc}{L}\begin{bmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix}$$

$$K^3 = \frac{EA}{L}\begin{array}{c} \begin{array}{cccc} x_1 & y_1 & x_4 & y_4 \end{array} \\ \begin{bmatrix} cs^2 & c^2s & -cs^2 & -c^2s \\ c^2s & c^3 & -c^2s & -c^3 \\ -cs^2 & -c^2s & cs^2 & c^2s \\ -c^2s & -c^3 & c^2s & c^3 \end{bmatrix} \end{array}$$

$$f^3 = T^{3T} \cdot \bar{f}^3 \longrightarrow \begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x4} \\ f_{y4} \end{bmatrix} = \begin{bmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{bmatrix} \begin{bmatrix} \bar{f}_{x1} \\ \bar{f}_{y1} \\ \bar{f}_{x4} \\ \bar{f}_{y4} \end{bmatrix}$$

Bar (1)



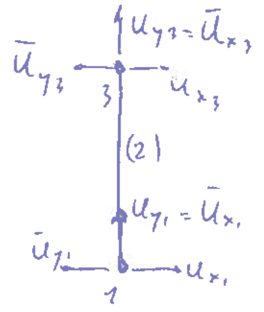
$$\bar{K}^1 = \frac{EAC}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad T^1 = \begin{bmatrix} -s & c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & c & s \end{bmatrix}$$

$$\begin{cases} \bar{u}_{x_1} = -s u_{x_1} + c u_{y_1} \\ \bar{u}_{y_1} = c u_{x_1} + s u_{y_1} \end{cases} \qquad \begin{cases} \bar{u}_{x_2} = -s u_{x_2} + c u_{y_2} \\ \bar{u}_{y_2} = c u_{x_2} + s u_{y_2} \end{cases}$$

$$\begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_2} \\ \bar{u}_{y_2} \end{bmatrix} = \begin{bmatrix} -s & c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & c & s \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \end{bmatrix}$$

$$K^1 = \frac{EAC}{L}\begin{bmatrix} -s & c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & c & s \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -s & c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & c & s \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} x_1 & y_1 & x_2 & y_2 \end{array} \\ \begin{bmatrix} cs^2 & -c^2s & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c \\ -cs^2 & c^2s & cs^2 & -c^2s \\ c^2s & -c^3 & -c^2s & c^3 \end{bmatrix} \end{array} \frac{EA}{L}$$

Bar (2)



$$\bar{K}^2 = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad T^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_3} \\ u_{y_3} \end{bmatrix}$$

$$\begin{cases} \bar{u}_{x_1} = u_{y_1} \\ \bar{u}_{y_1} = -u_{x_1} \end{cases} \qquad \begin{cases} \bar{u}_{x_3} = u_{y_3} \\ \bar{u}_{y_3} = -u_{x_3} \end{cases}$$

$$\begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_3} \\ \bar{u}_{y_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_3} \end{bmatrix}$$

$$K^2 = \frac{EA}{L}\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \frac{EA}{L}\begin{array}{c} \begin{array}{cccc} x_1 & y_1 & x_3 & y_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

1.-

$$\left.\begin{array}{l} K = K^1 + K^2 + K^3 \\ \bar{u} = \bar{u}^1 + \bar{u}^2 + \bar{u}^3 \end{array}\right\} \longrightarrow K \cdot \bar{u} = f$$

$$\frac{EA}{L}\begin{array}{c} \begin{array}{cccccccc} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{array} \\ \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symm} & & & & & & & c^3 \end{bmatrix} \end{array} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

~~La 5ª fila y columna corresponde al desplazamiento en x del nodo 3, a la matriz de rigidez~~

La 5ª fila y columna corresponde a la componente X del nodo 3. El cual será afectado por una fuerza nula, un desplazamiento nulo, y una deformación nula. El nodo 3 no está trabajando.

2.- Las condiciones de contorno son: $u_{x2} = u_{y2} = u_{x3} = u_{y3} = u_{x4} = u_{y4} = 0$

Las matrices quedan:

$$\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix} \quad \begin{cases} u_{x1} = \dfrac{H}{2cs^2} \\ u_{y1} = -\dfrac{P}{1+2c^3} \end{cases}$$

3.-

Si $\alpha = 0$ $\begin{cases} c = 1 \\ s = 0 \end{cases}$ $\longrightarrow$ Si $H \neq 0$ $\begin{cases} \text{El desplazamiento en } u_{x_1} = \frac{H}{0} \text{ (Indeterminado} \\ u_{y_1} = -\frac{P}{3} \end{cases}$

Si $\alpha = \frac{\pi}{2}$ $\begin{cases} c = 0 \\ s = 1 \end{cases}$ $\longrightarrow$ u_{x_1} vuelve a estar indeterminado

$u_{y_1} = -P$

Obviamente no tiene sentido físico que el ángulo ~~~~ $\alpha = 0$ ya que las barras (3) y (1) ocuparían la posición de la barra (1) y la estructura compresión

4.-

$$F^1 = \frac{EAc}{L} \cdot d^1, \quad F^2 = \frac{EA}{L} \cdot d^2, \quad F^3 = \frac{EAc}{L} \cdot d^3$$

$$\bar{u}^1 = T^1 u^1 \longrightarrow \begin{cases} \bar{u}^1_{x_1} = -\frac{H}{2cs} - \frac{Pc}{1+2c^3} \\ \bar{u}^1_{x_2} = 0 \end{cases}$$

$$\bar{u}^2 = T^2 u^2 \longrightarrow \begin{cases} \bar{u}^2_{x_1} = -\frac{P}{1+2c^3} \\ \bar{u}^2_{x_3} = 0 \end{cases}$$

$$\bar{u}^3 = T^3 u^3 \longrightarrow \begin{cases} \bar{u}^3_{x_1} = \frac{H}{2cs} - \frac{Pc}{1+2c^3} \\ \bar{u}^3_{x_4} = 0 \end{cases}$$

$$d_1 = \bar{u}^1_{x_2} - \bar{u}^1_{x_1} = \frac{H}{2cs} + \frac{Pc}{1+2c^3} \longrightarrow F^1 = \frac{EA}{L}\left(\frac{H}{2s} + \frac{Pc^2}{1+2c^3}\right)$$

$$d_2 = \bar{u}^2_{x_3} - \bar{u}^2_{x_1} = +\frac{P}{1+2c^3} \longrightarrow F^2 = \frac{EA}{L}\left(\frac{P}{1+2c^3}\right)$$

$$d_3 = \bar{u}^3_{x_4} - \bar{u}^3_{x_1} = -\frac{H}{2cs} + \frac{Pc}{1+2c^3} \longrightarrow F^3 = \frac{EA}{L}\left(-\frac{H}{2s} + \frac{Pc^2}{1+2c}\right)$$

Si $\alpha \to 0$ y $H \neq 0$ $\begin{cases} c = 1 \quad s = 0 \end{cases}$ se vuelve a crear una indeterminación en el miembro

$$\frac{H}{2s} = \frac{H}{0}$$