Computational Structural Mechanics & Dynamics

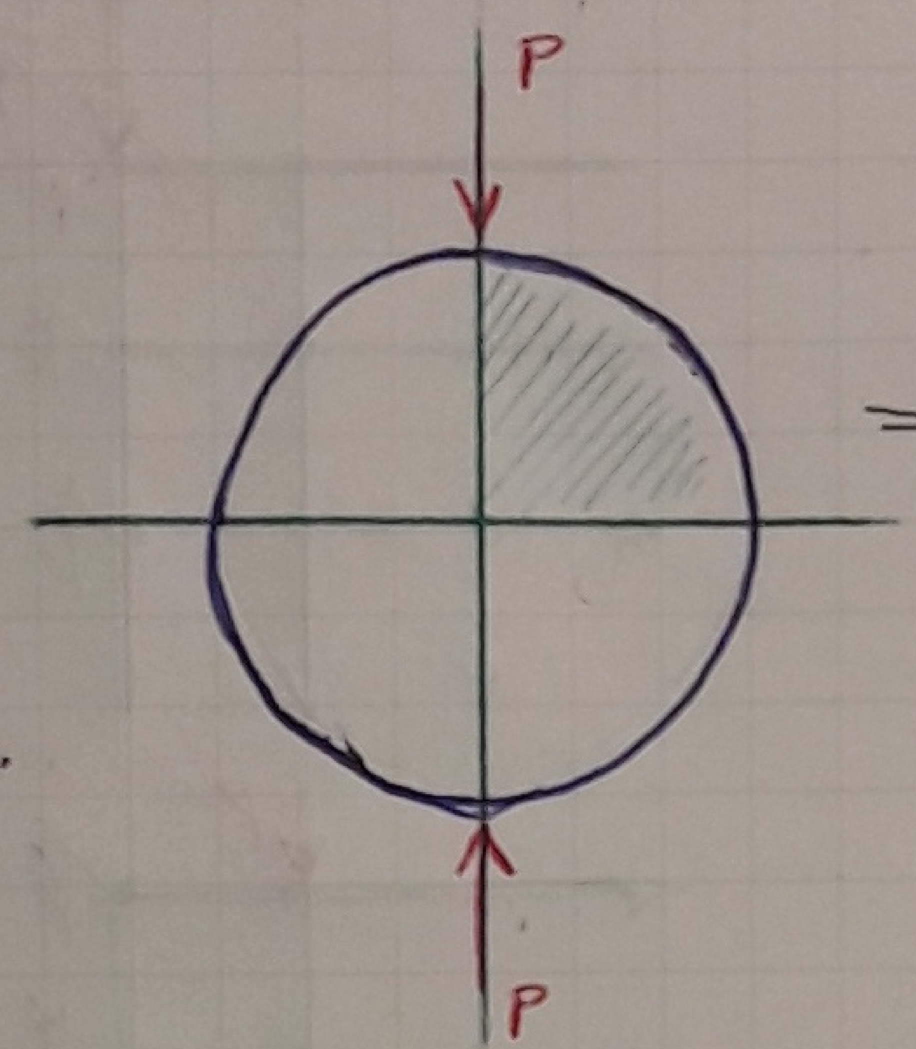
M. Mohsen Zadehkamand

Assignment 2.1 - 2.2 - 2.3

M.Sc. Computational Mechanics (2017) (19 Feb 2017)

Assignment 2.1

Sym.



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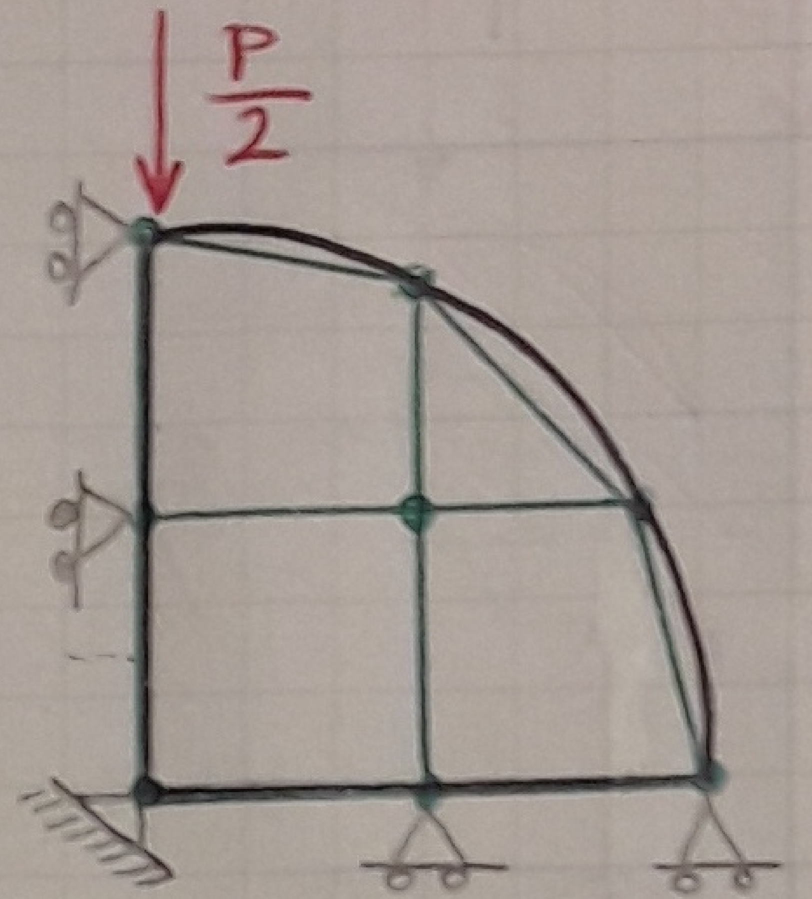
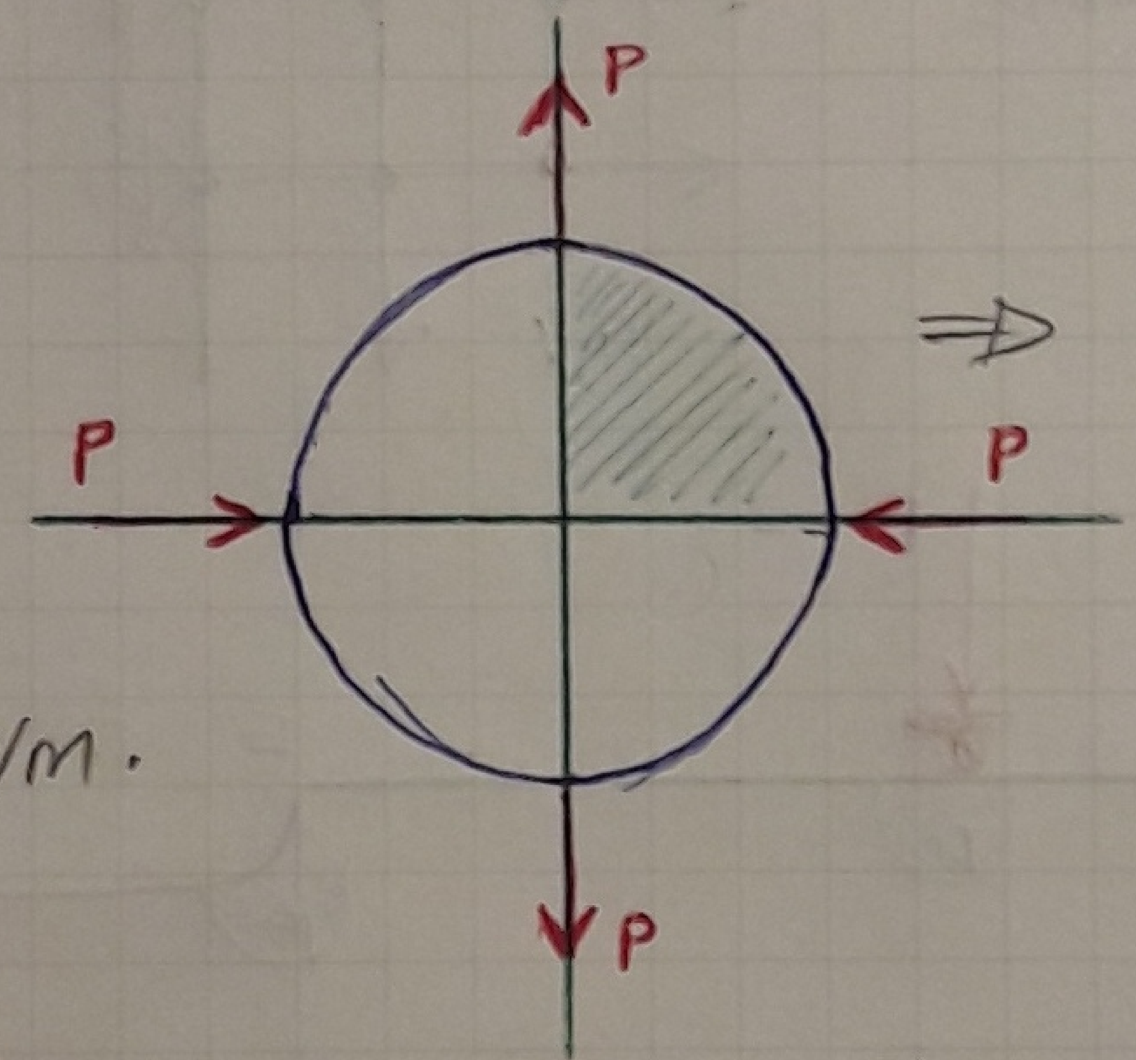


1/4 quarter

Sym.



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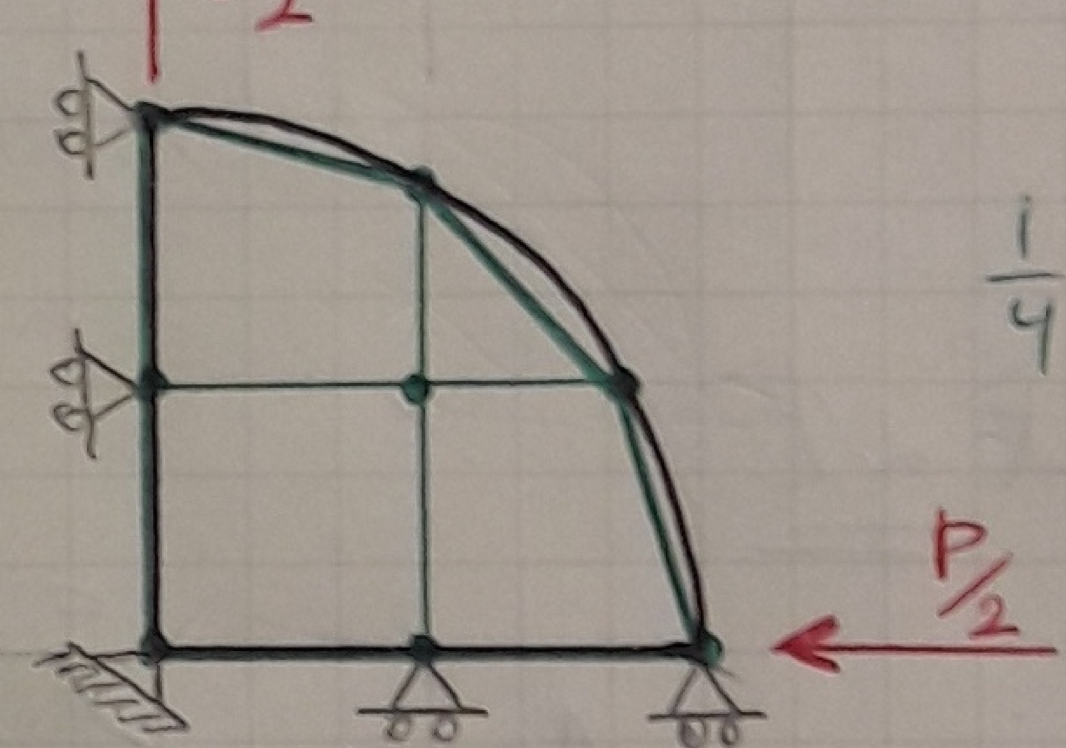
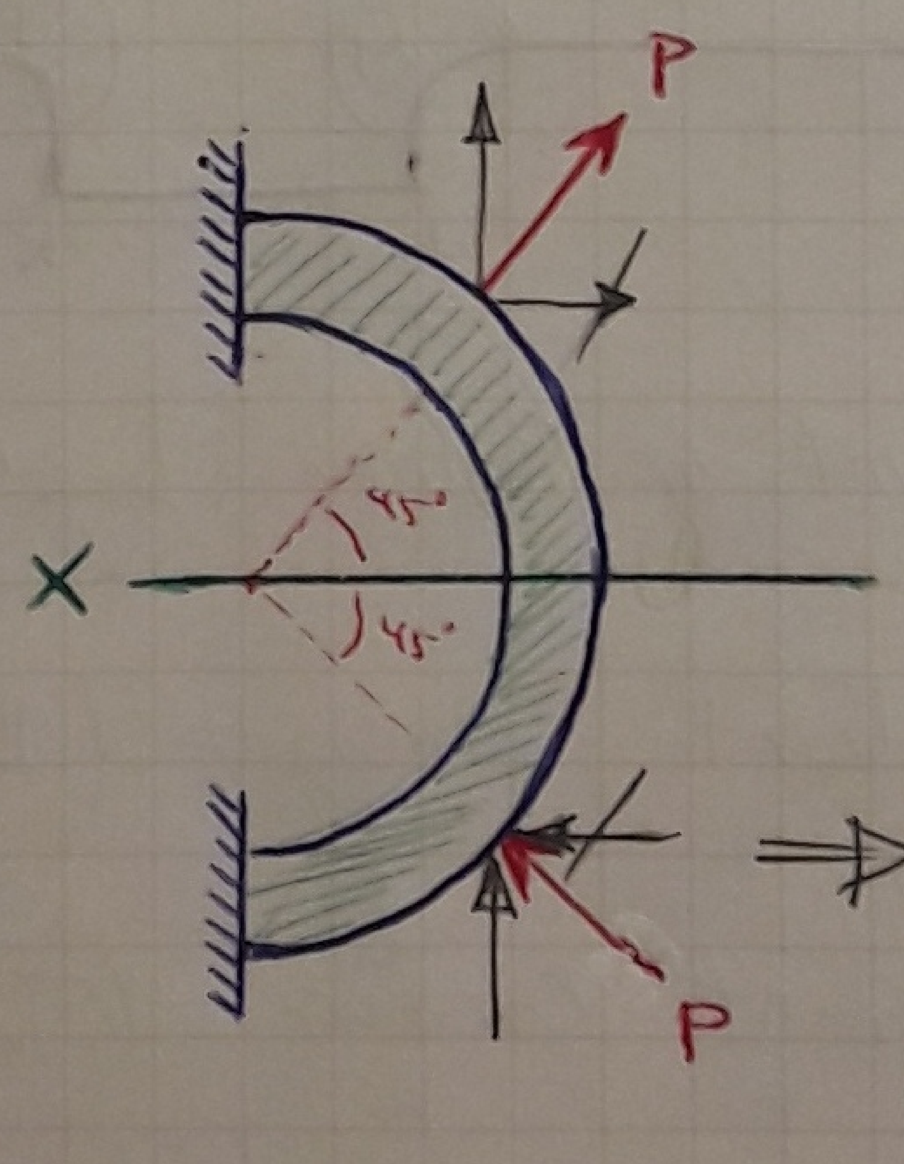


1/4 quarter

Anti Sym.



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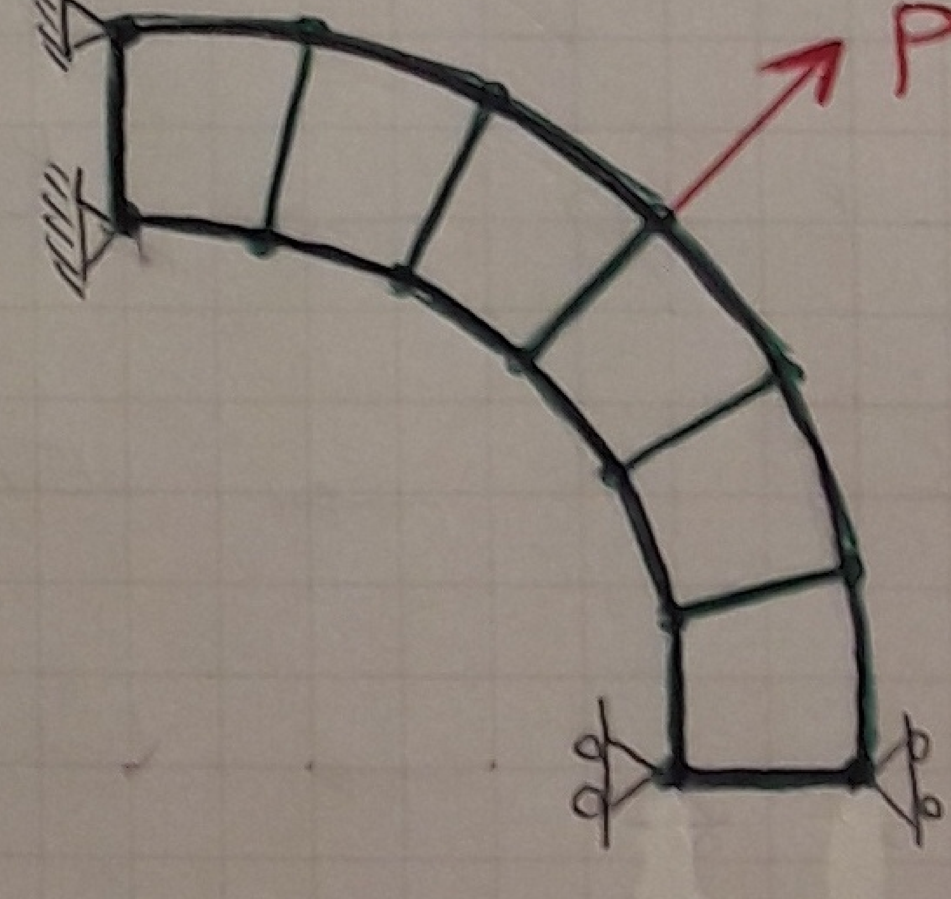
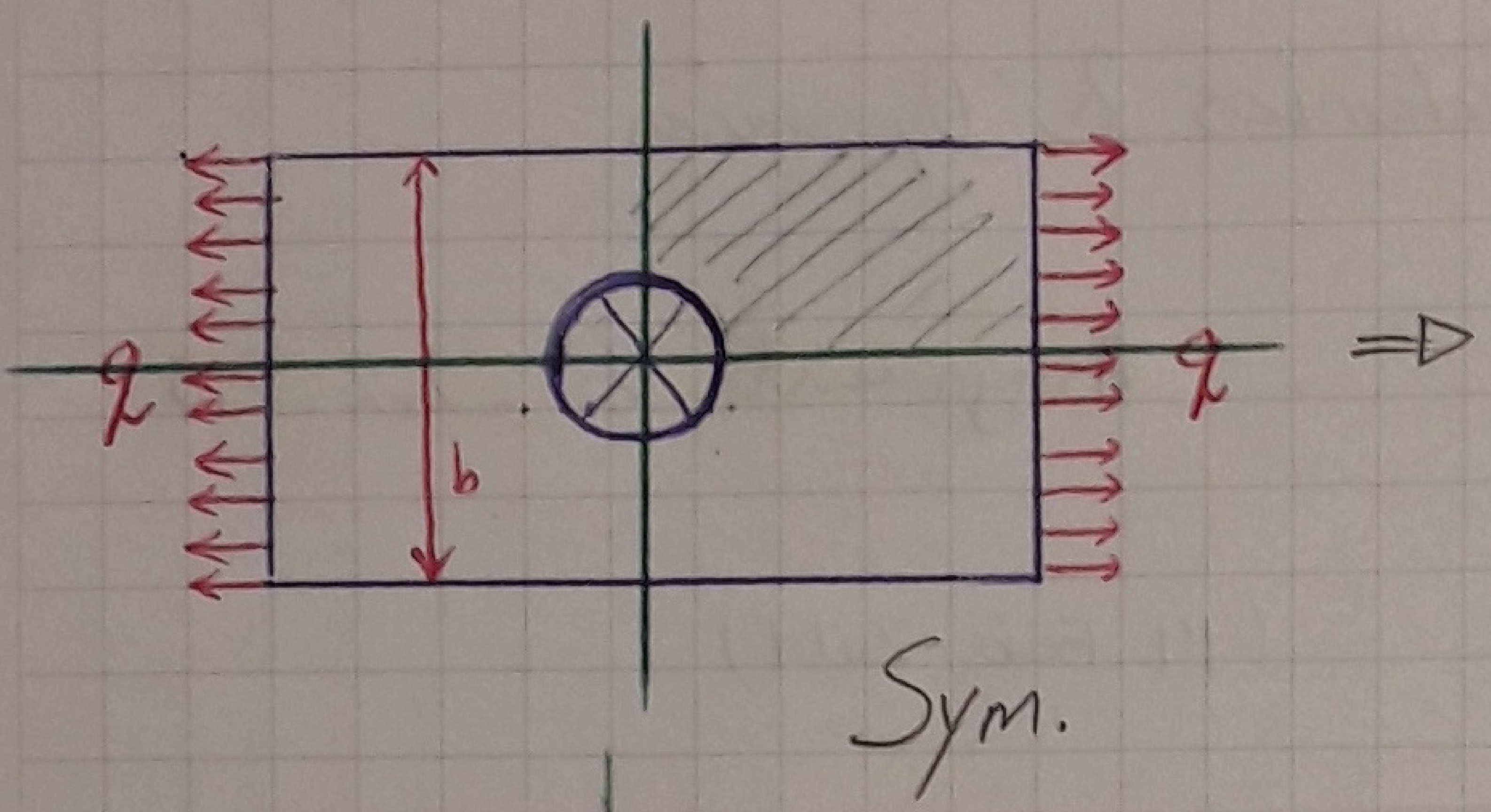
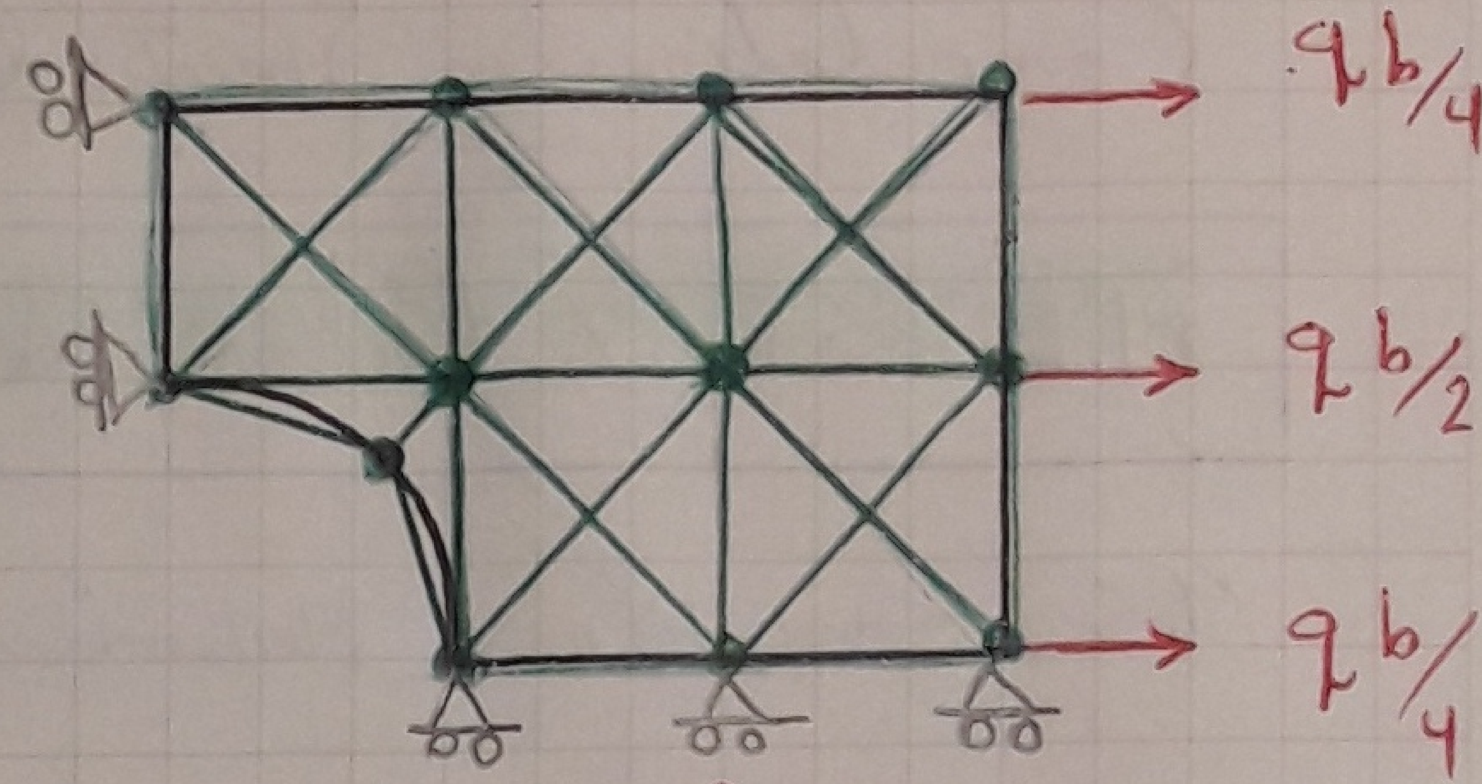
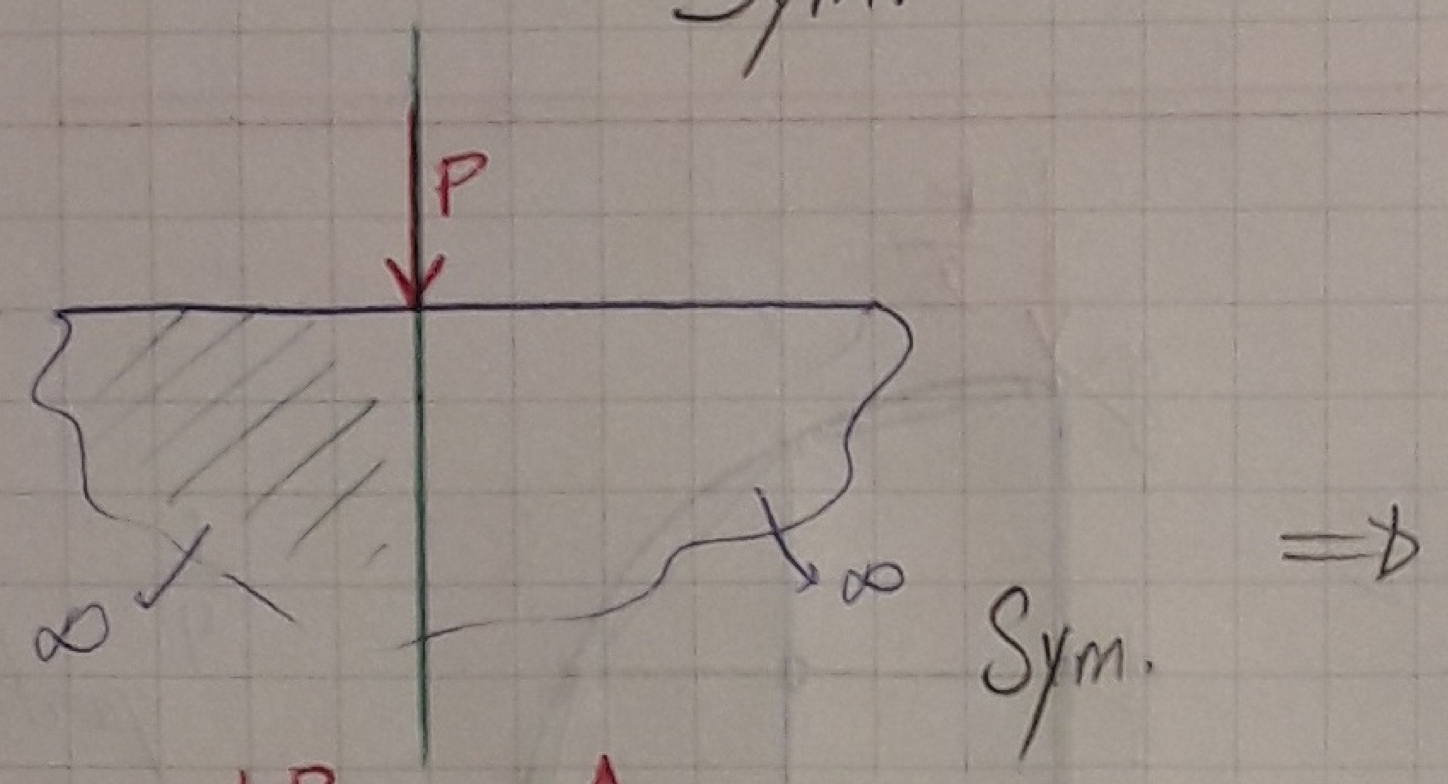
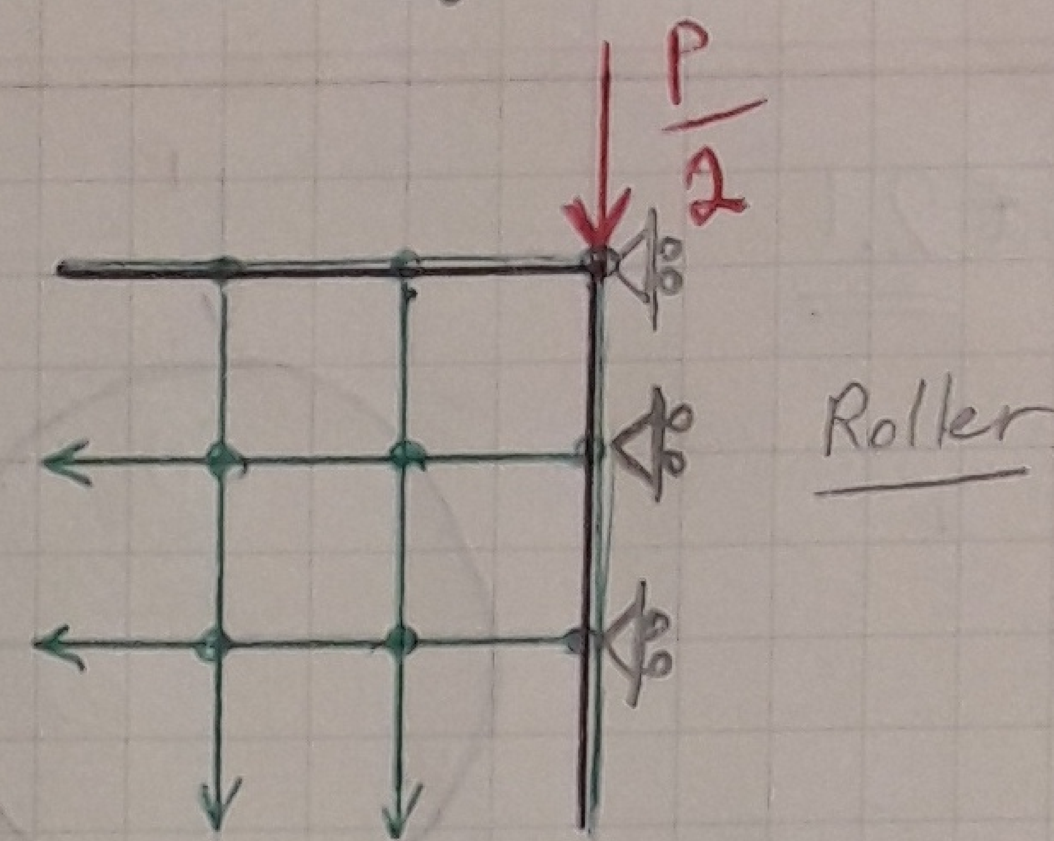


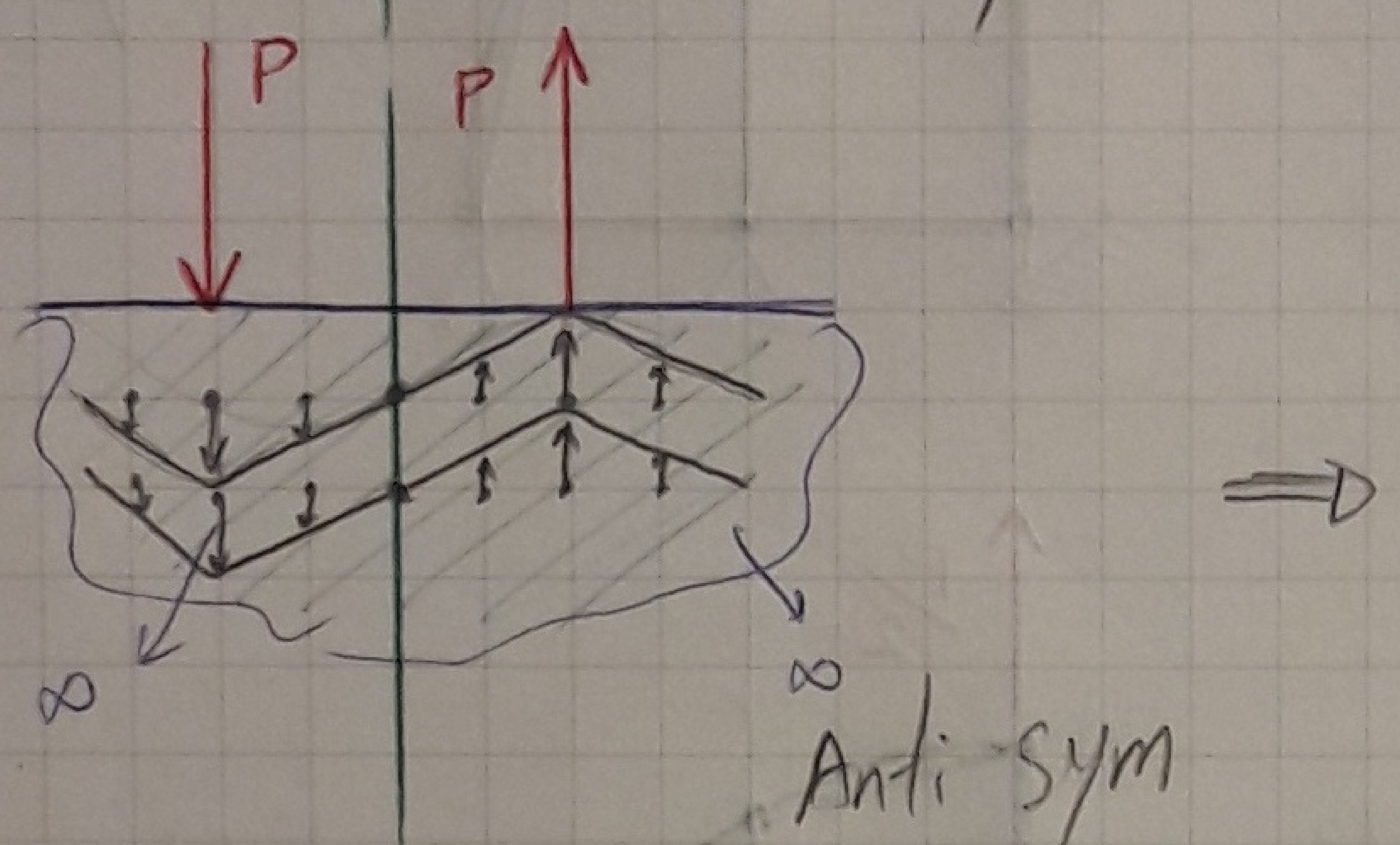
1/2 half

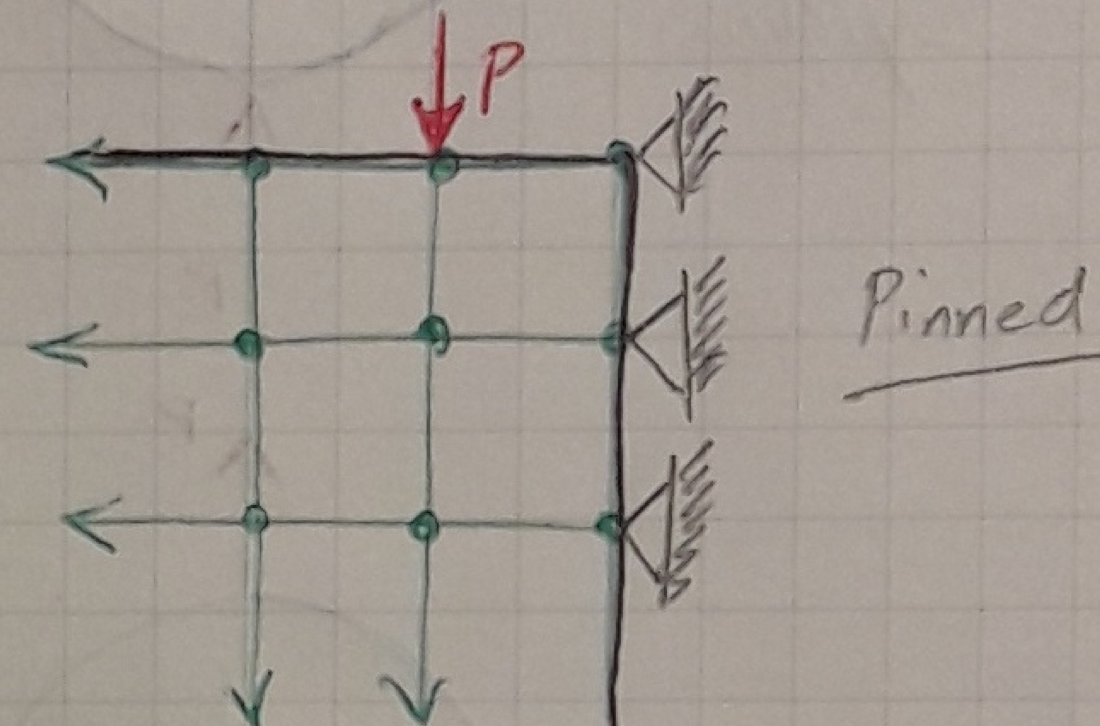




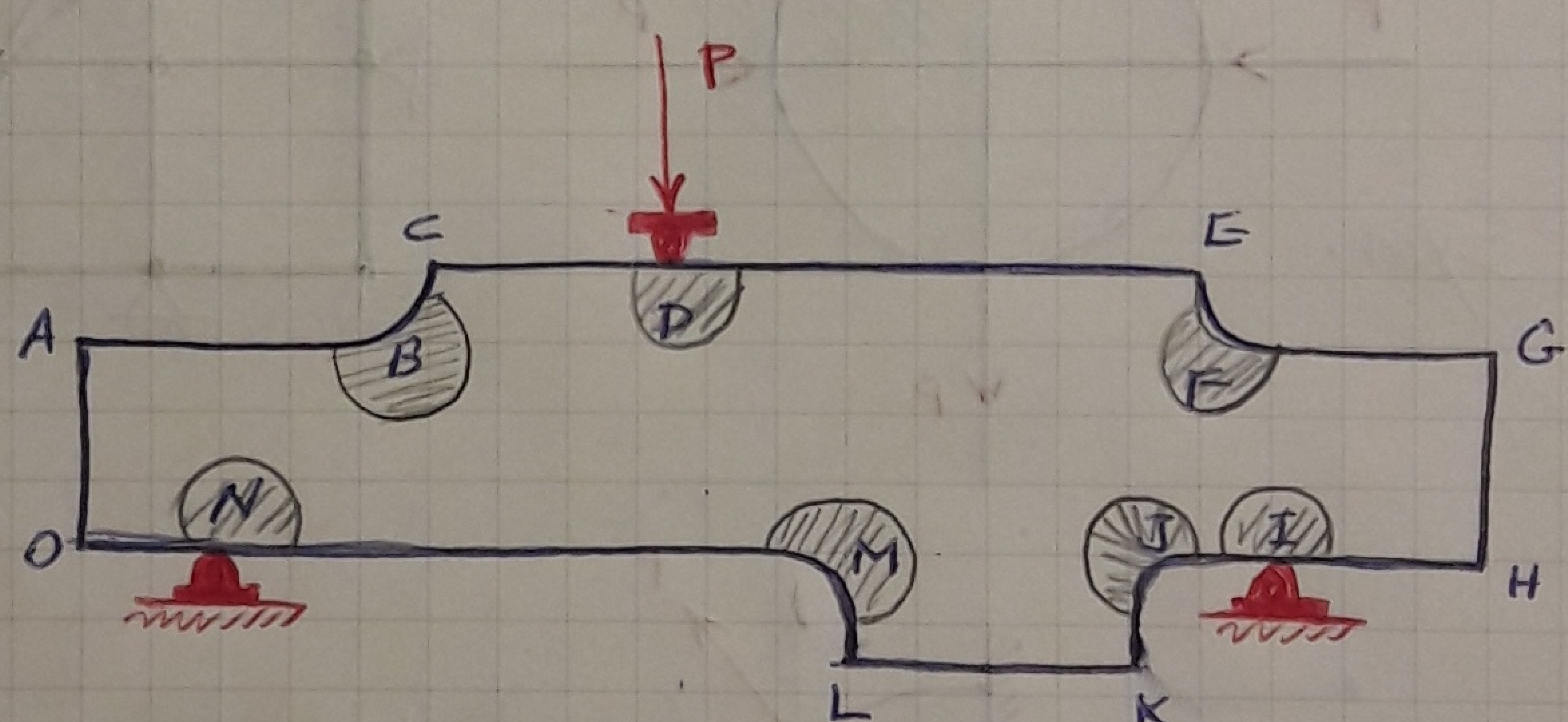








Assignment 2.2

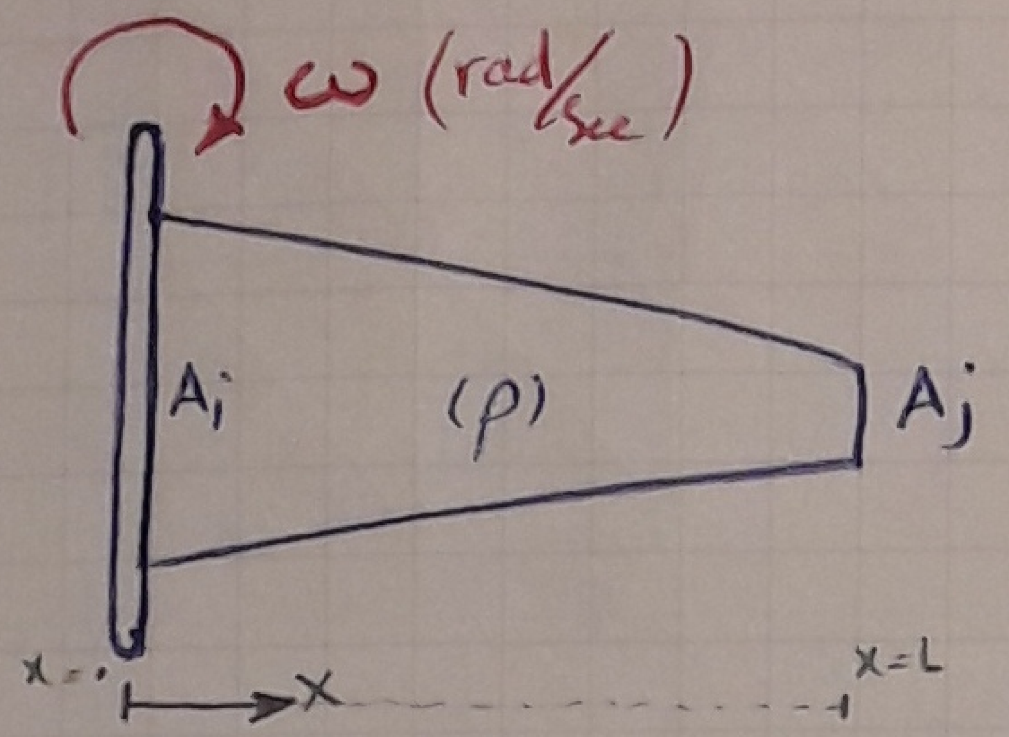


N – I – D → Point or Restrain Loading (consentrated)

M – J → Entrant Corners (Bot cord)

B – F → Entrant Corners (Top cord)

Assignment 2-3   Variational Formulation.



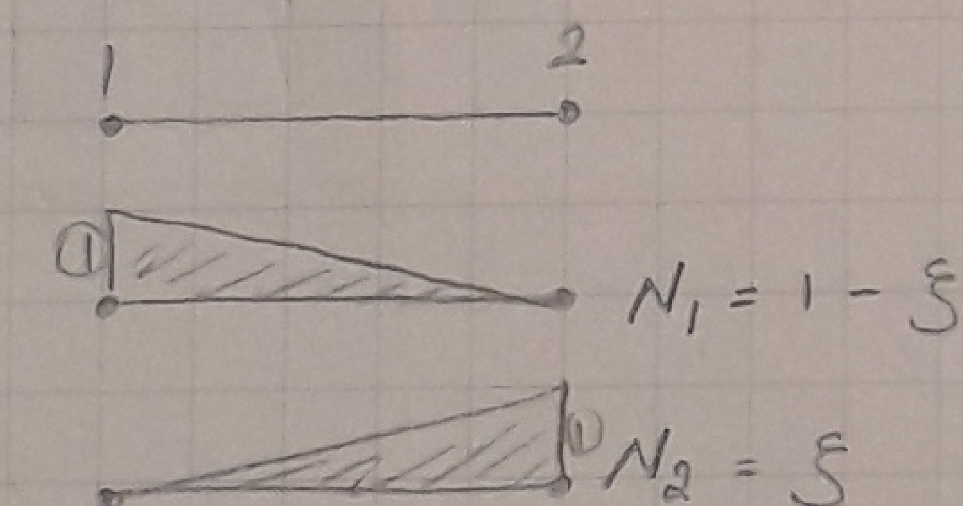
$$A = A_i(1-\xi) + A_j\,\xi$$

$$q(x) = \rho A \omega^2 x$$

$$W = \int_0^L q\,u\,dx$$

$$W^e = \int_0^1 q\,(N^T u^e)(l\,d\xi)$$

$$W^e = (u^e)^T \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\,d\xi$$



$$\xi = \frac{x - x_1}{l} = \frac{x}{l}$$

$$\Rightarrow f_{ext} = \int_0^1 \rho\big(A_i(1-\xi) + A_j\xi\big)\omega^2\,\xi l \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\,d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i\,\xi(1-\xi)^2 + A_j\,\xi^2(1-\xi) \\ A_i\,\xi^2(1-\xi) + A_j\,\xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i(\xi^3 + \xi - 2\xi^2) + A_j(\xi^2 - \xi^3) \\ A_i(\xi^2 - \xi^3) + A_j\,\xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \begin{bmatrix} A_i\left(\xi^4/4 + \xi^2/2 - \tfrac{2}{3}\xi^3\right) + A_j\left(\xi^3/3 - \xi^4/4\right) \\ A_i\left(\xi^3/3 - \xi^4/4\right) + A_j\,\xi^4/4 \end{bmatrix}_0^1$$

$$= \rho\omega^2 l^2 \begin{bmatrix} A_i\left(\tfrac{1}{4} + \tfrac{1}{2} - \tfrac{2}{3}\right) + A_j\left(\tfrac{1}{3} - \tfrac{1}{4}\right) \\ A_i\left(\tfrac{1}{3} - \tfrac{1}{4}\right) + A_j\,\tfrac{1}{4} \end{bmatrix} = \rho\omega^2 l^2 \begin{bmatrix} \tfrac{1}{12}A_i + \tfrac{1}{12}A_j \\ \tfrac{1}{12}A_i + \tfrac{1}{4}A_j \end{bmatrix}$$

$$= \frac{1}{12}\rho\omega^2 l^2 \begin{bmatrix} A_i + A_j \\ A_i + 3A_j \end{bmatrix}$$

if $A = A_i = A_j \longrightarrow f_{ext} = \begin{bmatrix} f_0 \\ f_1 \end{bmatrix} = \frac{1}{12}\rho\omega^2 l^2 \begin{bmatrix} 2A \\ 4A \end{bmatrix} = \frac{1}{6}\rho\omega^2 l^2 \begin{bmatrix} A \\ 2A \end{bmatrix}$