3.1

Plain strain

$$\left.\begin{aligned} e_{xx} &= \frac{1}{E}\left(\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right) \\ e_{yy} &= \frac{1}{E}\left(\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right) \\ e_{zz} &= \frac{1}{E}\left(\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right) \end{aligned}\right\}$$

Si $e_{zz} = 0 \rightarrow \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$ ①

$\cancel{\rightarrow e_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy})}$ ②

$\cancel{\rightarrow e_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx})}$ ③

sustituyendo ① en e_{xx}

$$e^{①}_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\left(\sigma_{yy} + \nu(\sigma_{xx} + \sigma_{yy})\right)\right) = \frac{1}{E}\left(\sigma_{xx}(1-\nu^2) - \sigma_{yy}\,\nu(1-\nu)\right)$$

$$= \frac{1-\nu^2}{E}\left(\sigma_{xx} - \sigma_{yy}\frac{\nu(1-\nu)}{1-\nu^2}\right) = \frac{(1-\nu^2)}{E}\left(\sigma_{xx} - \sigma_{yy}\left(\frac{\nu}{1+\nu}\right)\right)$$

$$\left[e^{①}_{xx} = e_{xx}\right] \left\{\begin{aligned} E &= E^*(1-\nu^2) \rightarrow E^* = \frac{E}{1-\nu^2} \\ \nu &= \nu^*(1+\nu) \rightarrow \nu^* = \frac{\nu}{1+\nu} \end{aligned}\right.$$

from plain strain:

$$\sigma_{xx} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(e_{xx} + \frac{\nu}{1-\nu}e_{yy}\right)$$

$$\sigma_{yy} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\frac{\nu}{1-\nu}e_{xx} + e_{yy}\right) \rightarrow e^{(2)}_{xx} = -\frac{\nu(1+\nu)}{E}\left(\sigma_{yy} - \frac{1-\nu}{\nu}\sigma_{xx}\right)$$

$$\left[e^{(2)}_{xx} = e_{xx}\right] \left\{\begin{aligned} E^* &= -\frac{E}{\nu(1+\nu)} \\ \nu^* &= \frac{1-\nu}{\nu^2} \end{aligned}\right.$$

b)

- $U = \frac{1}{2} e^T E e$

$U = \frac{1}{2}\left[t_{xx} e_{xx} + t_{yy} e_{yy} + 2 t_{xy} e_{xy}\right]$

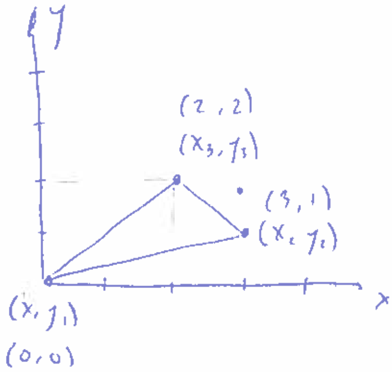
$t = E \cdot e \rightarrow \begin{bmatrix} t_{xx} \\ t_{yy} \\ t_{zz} \end{bmatrix} = E \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}$

$U = \frac{1}{2} e^T \cdot t = \frac{1}{2}\left[t_{xx} e_{xx} + t_{yy} e_{yy} + 2 t_{xy} e_{xy}\right] = \frac{1}{2} e^T E e$

- $U = \frac{1}{2} t^T E^{-1} t = \frac{1}{2} t^T e = \frac{1}{2}\left[t_{xx} e_{xx} + t_{yy} e_{yy} + 2 t_{xy} e_{xy}\right]$

3.2

1.-



$$E = \begin{bmatrix} 100 & 25 & 0 \\ 25 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

$$K^e = B^T E B h A^e$$

$$A^e = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 2$$

$$B = \frac{1}{2A} \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$\begin{cases} y_{23} = y_2 - y_3 = 1 - 2 = -1 \\ y_{31} = y_3 - y_1 = 2 - 0 = 2 \\ y_{12} = y_1 - y_2 = 0 - 1 = -1 \end{cases} \qquad \begin{cases} x_{32} = x_3 - x_2 = 2 - 3 = -1 \\ x_{13} = x_1 - x_3 = 0 - 2 = -2 \\ x_{21} = x_2 - x_1 = 3 - 0 = 3 \end{cases}$$

$$B = \frac{1}{4} \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 3 \\ -1 & -1 & -2 & 2 & 3 & -1 \end{bmatrix}$$

$$K^e = \frac{Ah}{4A^2} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & -2 \\ 0 & -2 & 2 \\ -1 & 0 & 3 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} 100 & 25 & 0 \\ 25 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 3 \\ -1 & -1 & -2 & 2 & 3 & -1 \end{bmatrix}$$

$$K^e = \begin{bmatrix} 18,75 & 9,38 & -12,50 & -6,25 & -6,25 & -3,13 \\ 9,38 & 18,75 & 6,25 & 12,50 & -15,63 & -31,25 \\ -12,50 & 6,25 & 75 & -37,50 & -6,25 & 31,25 \\ -6,25 & 12,5 & -37,5 & 75 & 43,25 & -87,50 \\ -6,25 & -15,63 & -62,50 & 43,25 & 68,75 & -28,13 \\ -3,13 & -31,25 & 31,25 & -87,50 & -28,13 & 118,75 \end{bmatrix}$$

Al sumar las filas ~~o~~ columnas en 1, 3, 5 o 2, 4, 6 se comprueba que el resultado es cero.

Las filas y columnas 1, 3, 5 representan la rigidez correspondiente al eje X de los nodos. Mientras que las filas y columnas 2, 4, 6 corresponden al eje Y.

Si la matriz de rigidez relaciona fuerzas y desplazamientos y estos son nulos a lo largo de los ejes en los nodos, quiere decir que el triángulo no se deforma y los nodos tienen siempre la misma posición relativa entre ellos