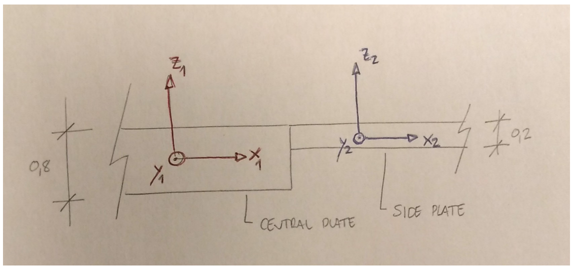
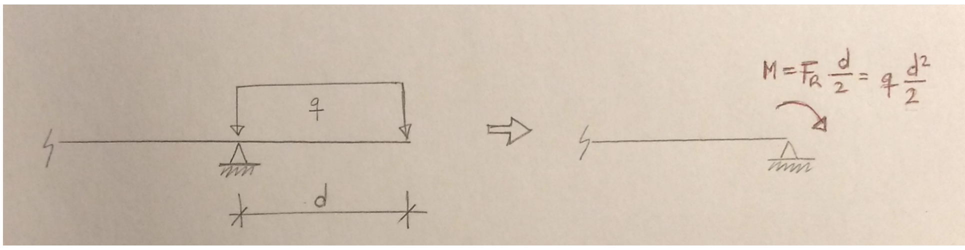
Exercise a

The structure of the proposed exercise consists of a square plate of 0.8m thickness with four laterally attached rectangular cantilever plates of 0.2m thick.

When studying the structure as a whole, the main difference between the structures of the first and second sections appears. In the first one, the plane that has in common the five elements of the system is the upper one. Therefore, due to the different thicknesses, the x-y axes of each of the pieces are not coplanar, which implies that the central element must be studied in first place and the lateral ones in second.

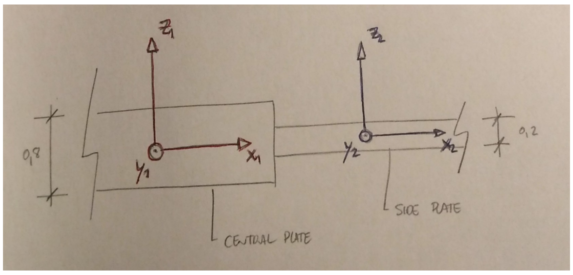


In order to do this each of the lateral plates will be transformed into a boundary condition located along the corresponding side of the central plate. This condition will consist in a rotation momentum resulting from multiplying the resultant force on the cantilever (the sum of the self-weight plus the surface forces and the possible point forces that might be applied) times the distance from its center of application to the edge of the central plate.



After analyzing the central plate, we can proceed with the side ones by transfering as boundary conditions the resultant values of the first analysis in the zone of contact between the two elements (vertical displacement and turns).

In the second section, however, the central planes for each element coincide. Therefore, we can solve the exercise at one time by setting the corresponding value of t (thickness) for each of the elements, depending on whether they belong to the central body, or to the sides ones.

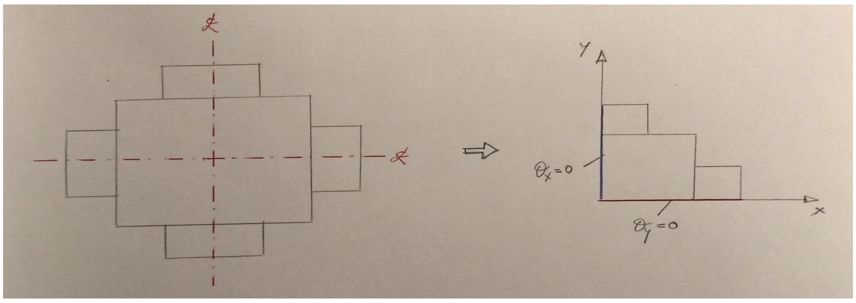


For the discretization of the problem some plate element will be used. Attending to the formula thickness (*t*)/ width (*w*) ≤ 0.10 to decide whether a plate is considered thick or thin, it can easily checked that both the central plate and the lateral parts would be enclosed into the second type. Thus, both MZC quadrangular elements (valid for thin plates) and quadrangular RM elements (valid for both thin and thick plates) might be used, because due to the geometry of the structure, it can be discretized into rectangular-shaped elements. If this were not the case, it would have been necessary to use triangular DK elements, for example.

In any case, the degrees of freedom to be studied in each of the mesh nodes are three: vertical displacement *w,* and turns in x-axis θ_x and y-axis θ_y.

Since surface of both structures have two axes of symmetry, the problem can be simplified and just the behavior of a quarter of the structures can be studied. By means of this procedure a smaller area will be covered and there is a possibility of getting a finer mesh at the same computational cost. The mesh must be denser in the singular areas of the structure, since it is at these places where there are more chances of problems to appear: contact zones between the plates and support zones in the case of the structure was supported on columns.

When setting up the boundary conditions, turns along the axes of symmetry will be restricted as follows:

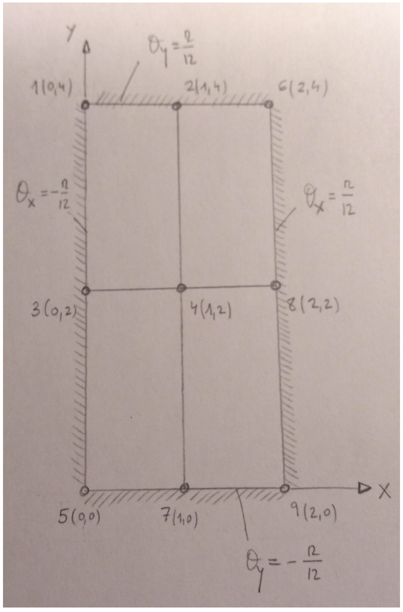


Besides, if there was any force of linear or punctual type on the structure, they would have to be divided by two (if it was located on one of the two axes of symmetry) or by four (if it was located on the central point) .

Since the statement does not provide any information in this respect, support conditions of the central plate might be clamped (with the three degrees of freedom restricted) or, more likely, simply supported both on punctual supports or linear supports. In any case, boundary conditions will be fixed by restricting the required degrees of freedom.

Exercise b

A patch test is defined on an MCZ element, consisting in the application of a zero vertical displacement as well as a rotation of Π/12 for each of the perimetral nodes of the following geometry, leaving the central node totally free:



Attending to the symmetry of the figure both in its geometry and its conditions, it is expected to obtain a rotation in x- and y-axis equal to zero at the central node, values that will confirm the approval or the fail of the patch test.

From the following system:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{f}$$

What differentiates one system from another will be the stiffness matrix **K**. Once this is obtained by means of the MATLAB routine *Plate_MZC*, for a given set of values of material properties (that will be the same for every tested geometry), the required results will be obtained for any case. Due to its large dimensions, the matrix **K** is not reproduced here.

For the imposed boundary conditions, the following vector **u** is obtained:

$$\mathbf{u} = [w_1 \;\; \theta_{x1} \; \theta_{y1} \;\; w_2 \;\; \theta_{x2} \, \theta_{y2} \ldots \ldots \ldots \; w_9 \;\; \theta_{x9} \, \theta_{y9}]^T$$

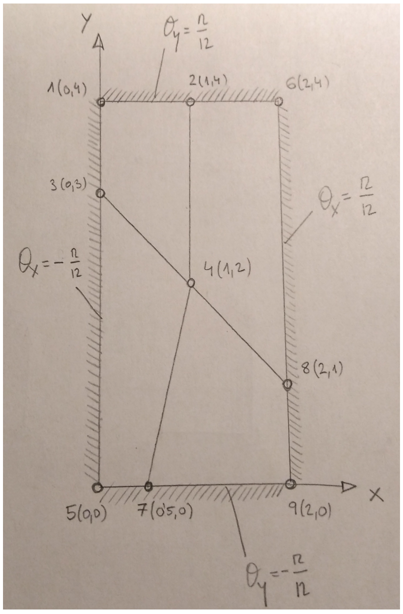
$$= [\,0 \;\, -\Pi/12 \;\, \Pi/12 \;\, 0 \;\, 0 \;\, \Pi/12 \;\, 0 \;\, -\Pi/12 \;\, 0 \;\, w_4 \;\, \theta_{x4} \, \theta_{y4} \;\, 0 \;\, -\Pi/12 \;\, -\Pi/12 \;\, 0 \;\, \Pi/12 \;\, \Pi/12 \;\, 0 \;\, 0 \;\, -\Pi/12 \;\, 0 \;\, \Pi/12 \;\, 0 \;\, 0 \;\, \Pi/12 \;\, -\Pi/12\,]^T$$

In the absence of external forces, the vector $\mathbf{f} = \mathbf{0}$.

Multiplying rows number 10, 11 and 12 by vector **u** and making it equal to zero, a three-equation system in terms of $w_4 \;\; \theta_{x4} \, \theta_{y4}$ is obtained. From there it can be easily checked that:

$$w_4 = -0.1860 \qquad \theta_{x4} = 0 \qquad \theta_{y4} = 0$$

It was expected to obtain values for the turns in the x- and y-axis equal to zero. Therefore, for a rectangular shape of MZC elements, patch test is passed. The problem is that some elements do not work properly for non-regular shapes, so the designed patch test would not be realiable. For that reason, now it is time to see what would happen if the patch test geometry was composed of non-rectangular quadrilateral elements, as follows:



Solving the system analogously with the same **u** and **f** vectors and a new **K** stiffness matrix, the obtanied results for the studied free node (number 4) are the following:

w_4 = -0.1361 θ_{x4} = -0.1284 θ_{y4} = 0.097

As in the previous case, it was expected to obtain results for the turns in x- and y-axis equal to zero. Thus, the second patch test is not passed for the studied element. As it was mentioned in the theoretical classes, the MZC element is not reliable in the case of non-rectangular forms.