

CSMD: ASSIGNMENT 8: PLATES

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a)

a.1)

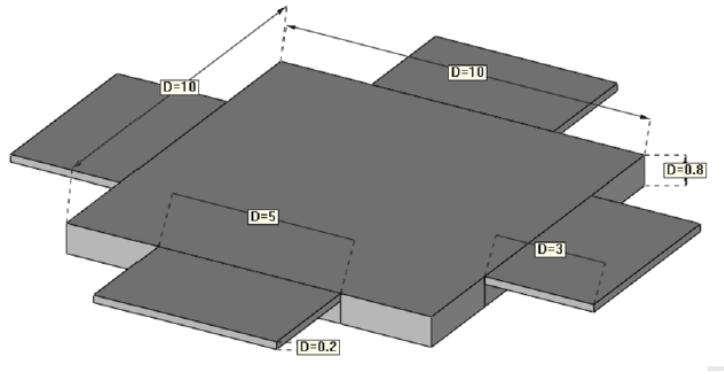


Figure 1: Problem 1

This problem consists of 5 different plates with different dimensions which are fitted together. If a bending load is assumed, plate's theory can be used to solve the problem instead of using 3D elasticity. This problem has a thick plate and four thin plates. The thick plate should be solved using Reissner-Midlin theory, while the thin plates can either be solved using Kirchoff-Love theory or Reissner-Midlin. If Reissner-Midlin is used everywhere, it may be necessary to use reduced integration or discrete Kirchoff elements in the thin plates to avoid shear-locking effects. If Kirchoff-Love theory is solved for in the thin plates, then the complications are in the transmission conditions at the boundary between the plates: An iterative scheme may be necessary to ensure the transmission conditions and the continuity of the shear force and bending moment, as well as displacement, across the boundary.

For example, we could solve the problem using quadrilateral Reissner-Midlin elements with a 2×2 integration rule for K_b and 1 Gauss point for K_s in the thin plates, and a 2×2 rule for both K_b and K_s for the thick plate.

However, if we were interested in the concentration of stresses at the joint between two different plates, it would be necessary to use 3D elasticity.

a.2)

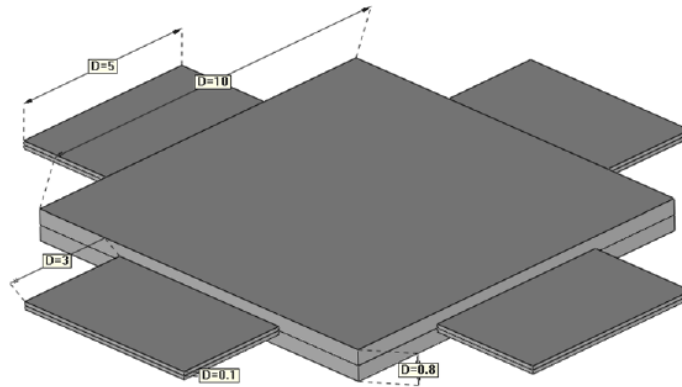


Figure 2: Problem 2

This problem is equal than problem a.1), but with parallel middle plane. Since in plates theory it is assumed that deflections and rotations are constant along the cross section, the same results as in a.1) will be obtained.

b)

The patch test has been performed using the patch of elements defined in figure 3a. For $t=0$ and $h=0$, rectangular elements are obtained (see figure 3b). The interior node has coordinates $x=0$, and $y=0$. The displacement used in the patch test is a quadratic one:

$$\begin{cases} w = 1 + x + y + x^2 + y^2 + xy \\ \frac{dw}{dx} = 1 + 2x + y \\ \frac{dw}{dy} = 1 + 2y + x \end{cases}$$

The boundary conditions are such that w , $\frac{dw}{dx}$ and $\frac{dw}{dy}$ are imposed at the exterior nodes. At the interior node ($x=0, y=0$) we should obtain the following results:

$$\begin{cases} w = 1 \\ \frac{dw}{dx} = 1 \\ \frac{dw}{dy} = 1 \end{cases}$$

The patch test has been performed for different shapes of the elements. When setting $t=0$ and $h=0$, the results obtained for different values of a and b (see figures 4 and 5) show that the results obtained at the inner node coincide with the prescribed displacement field. However, when we have $t \neq 0$ (figure 6) or $h \neq 0$ (figure 7), the results obtained at the inner node do not coincide with the displacement field imposed and the element fails the patch test. This happens because the MCZ element is a non-conforming element that converges to the correct solution only for rectangular shapes.

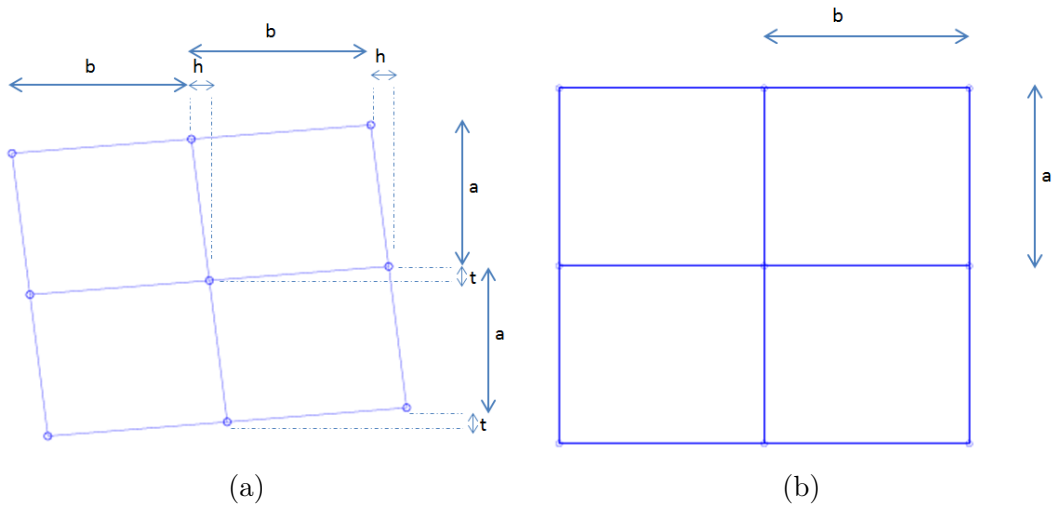


Figure 3

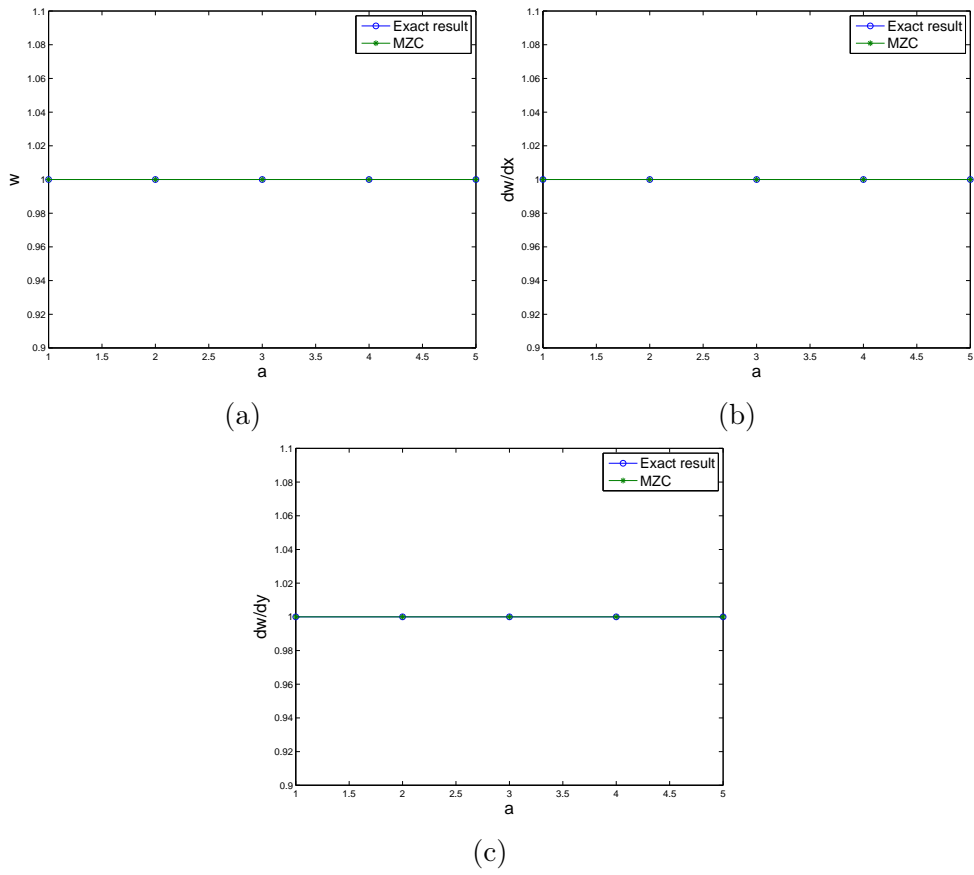


Figure 4: Results at the inner node for $b=1$ and $t=h=0$.

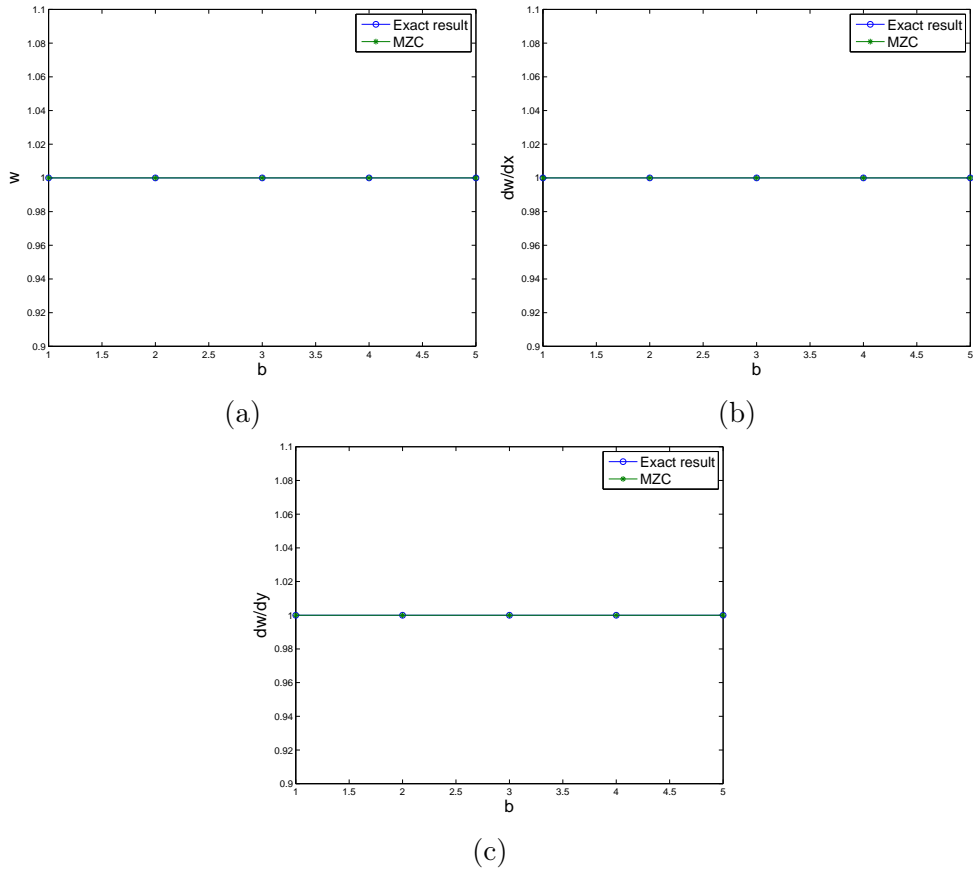


Figure 5: Results at the inner node for $a=1$ and $t=h=0$.

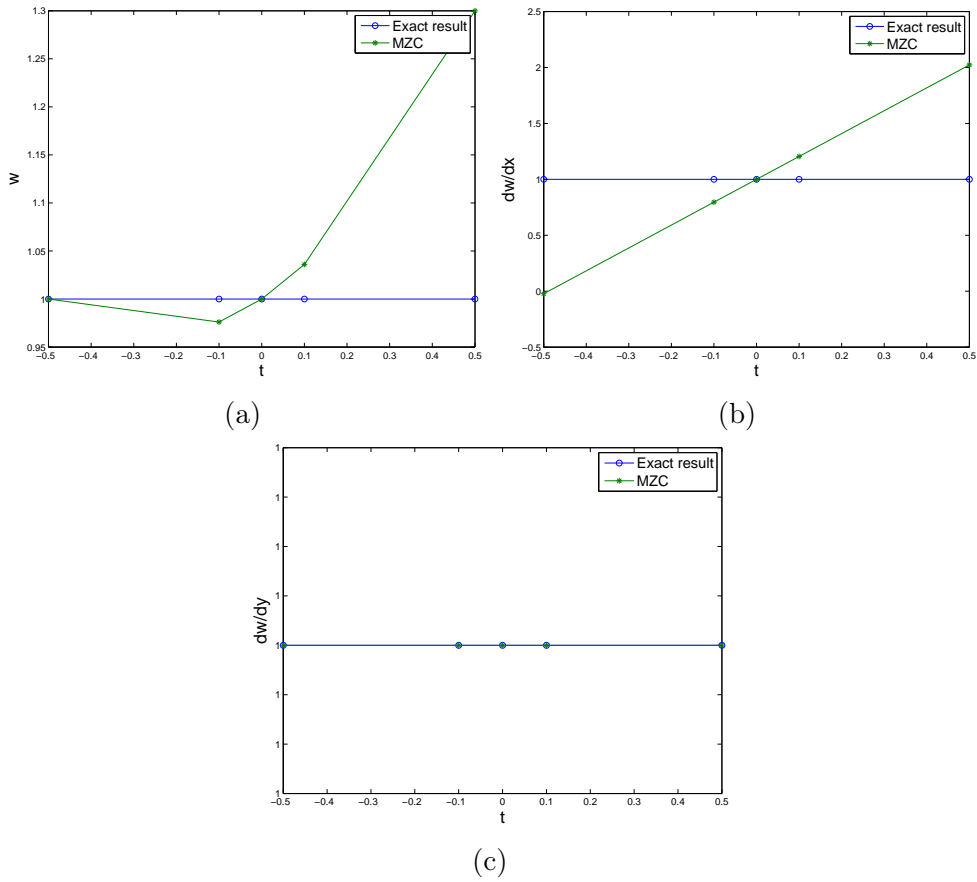


Figure 6: Results for at the inner node for $h=0$ and $a=b=1$.

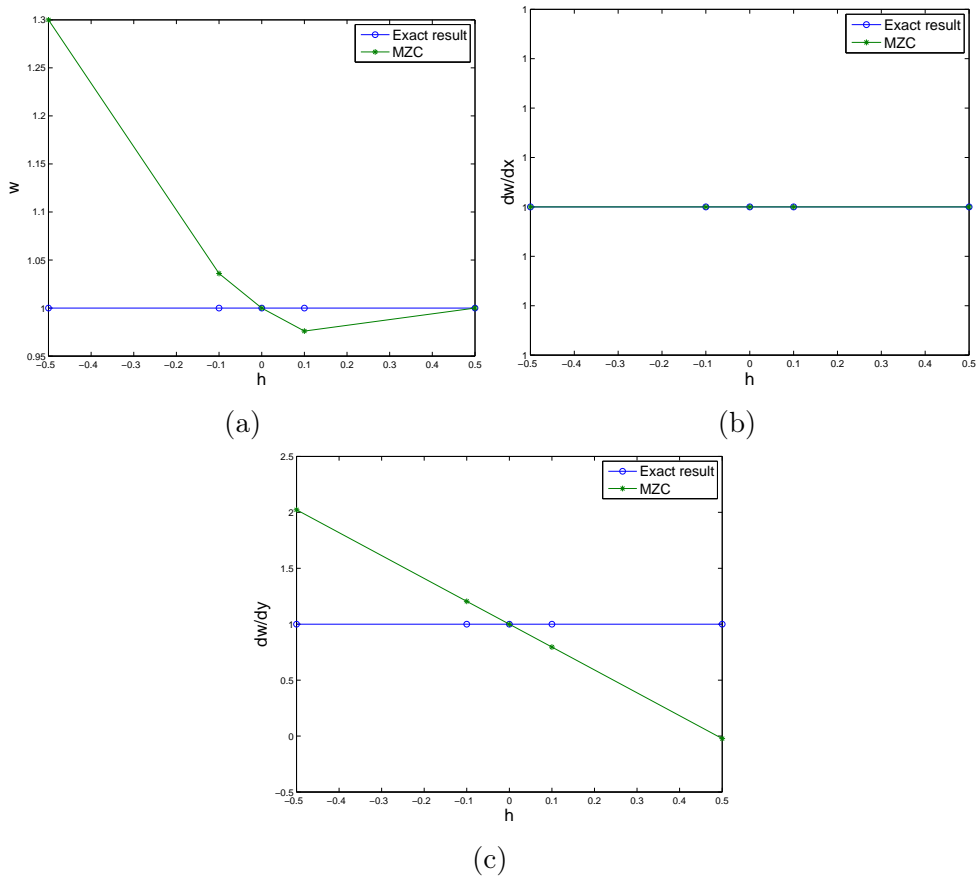


Figure 7: Results at the inner node for $t=0$ and $a=b=1$.