Computational Structural Mechanics and Dynamics **March 22, 2019**
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Assignment 6

6.1 Program In Matlab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix.

Many procedures to eliminate shear locking in Timoshenko beam elements have been proposed. Reduced integration is a popular method to reduce the influence of the transverse shear stiffness by under-integrating the terms in $K_s^{(e)}$ using a quadrature of one order less than is needed for exact integration. The terms of $K_b^{(e)}$ are still integrated exactly.

$$K_b^{(e)} = \left(\frac{\hat{D}_b}{l}\right)^{(e)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{1}$$

Where: $\hat{D}_b = EI_y$
For homogeneous material, the computation of $K_s^{(e)}$ with a single integration point gives:

$$K_s^{(e)} = \left(\frac{\hat{D}_s}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \cdots & 1 & -\frac{l^{(e)}}{2} \\ symm. & & \cdots & \frac{(l^{(e)})^2}{4} \end{bmatrix} \tag{2}$$

Where: $\hat{D}_s = GA^*$

6.2 Discussion of the results.

In the following graphs the three methods will be compared by plotting the maximum moment, maximum shear and maximum displacement values for each of the a/L ratios.
L will always have the same value (4m) and a will take the following values:

$$a = 0{,}001$$
$$a = 0{,}005$$
$$a = 0{,}010$$
$$a = 0{,}020$$
$$a = 0{,}050$$
$$a = 0{,}100$$
$$a = 0{,}200$$
$$a = 0{,}400$$

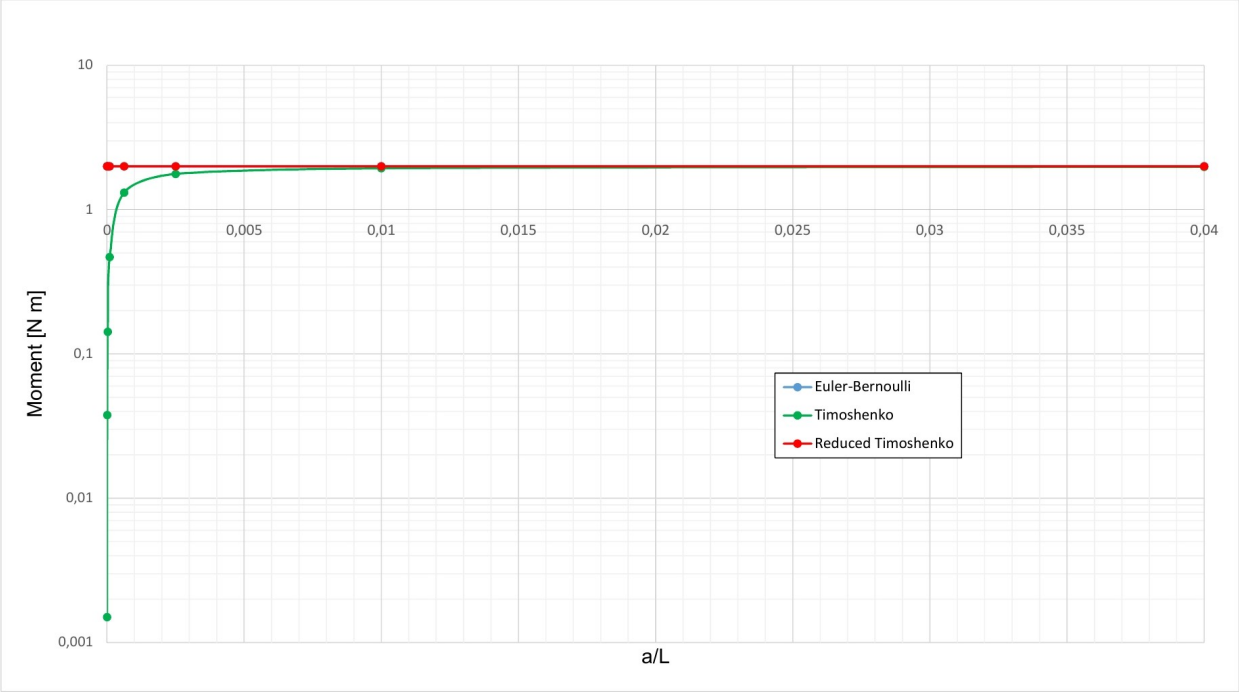

Figure 6.2.1: Maximum moment

Comparing the moment efforts it is possible to observe how for Timoshenko and a low value of the relation a/L, the value has a marked difference with the two remaining methods. When the value of the relation a/L is greater, these three methods are equalized.

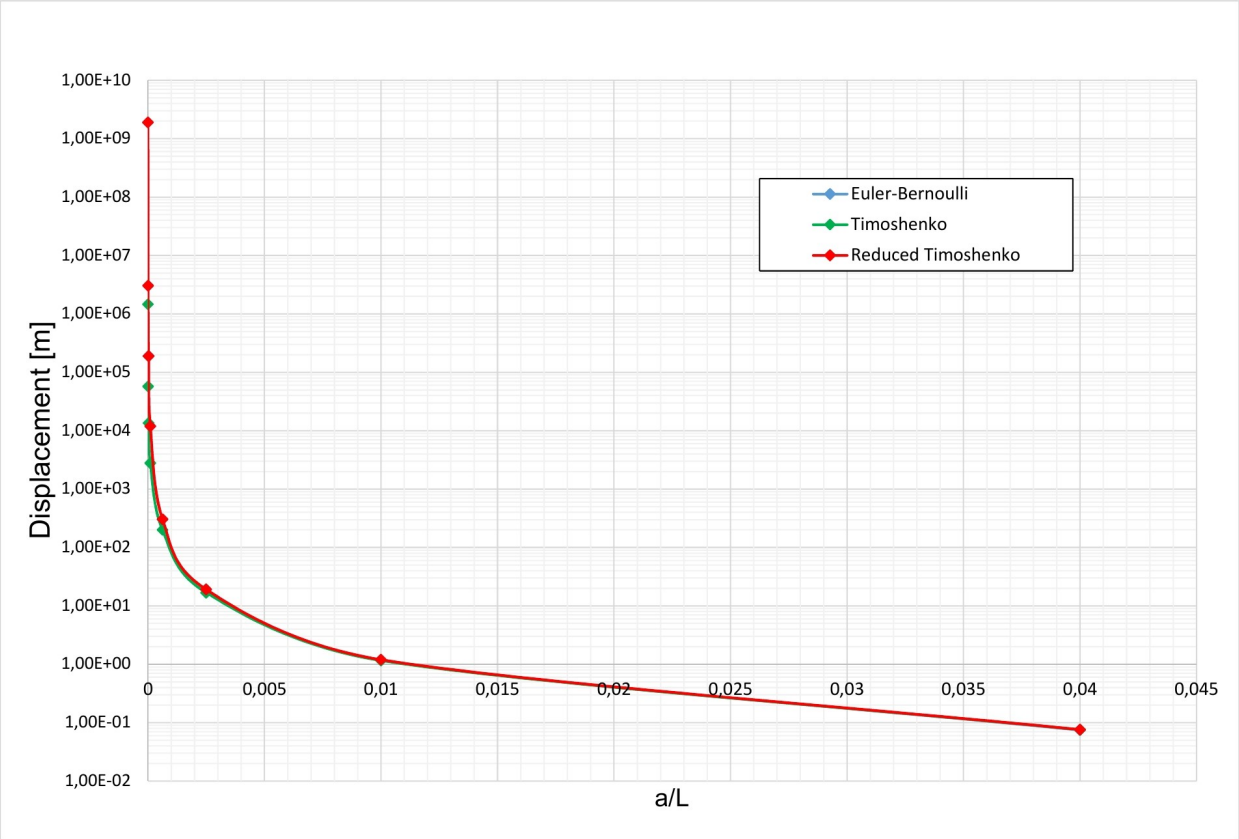

Figure 6.2.2: Maximum displacement

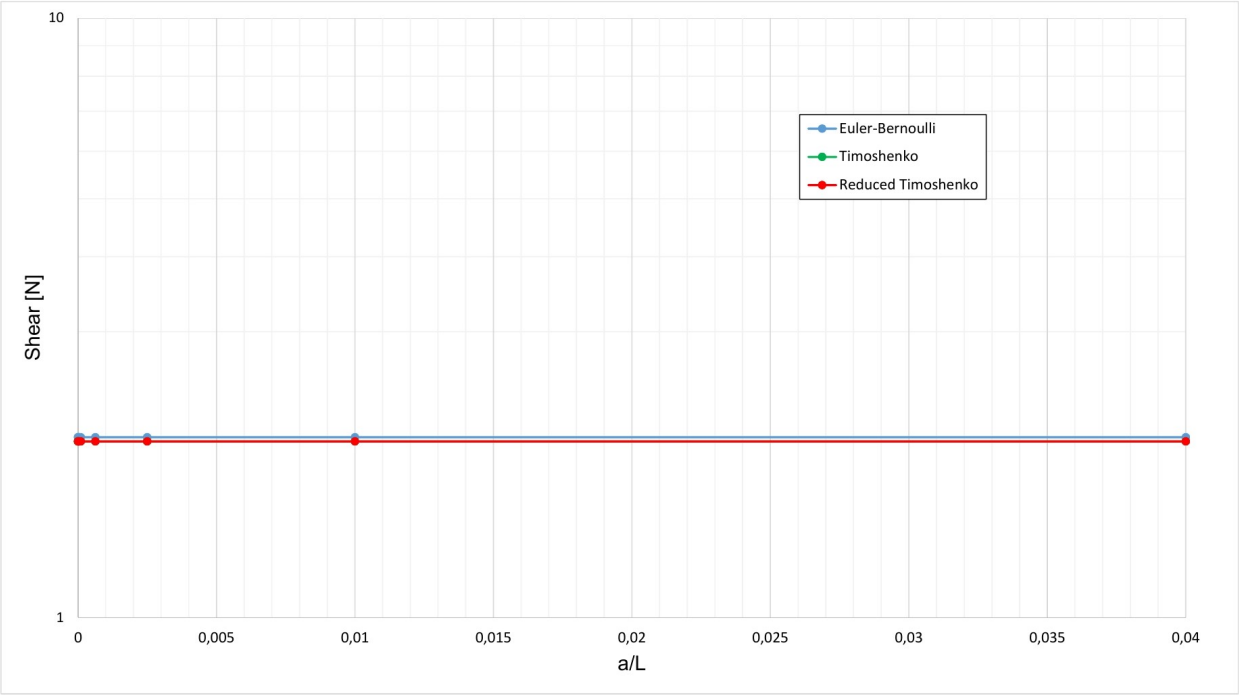

Figure 6.2.3: Maximum shear

Finally, for the maximum values of shear and displacement for each of the relationships, the three methods have a similar behavior.
Euler-Bernoulli has a small difference with Timoshenko and Reduced Timoshenko in the case of the shear effort while in the case of maximum displacement Reduced Timoshenko is a bit separated from the two remaining methods.