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Master of Science in Computational Mechanics

Assignment 7- BEAMS

a) For a simply supported beam, Timoshenko 2 Nodes Beam element with reduced integration for the shear stiffness matrix is programmed in Matlab by changing the K matrix by following matrix and changing gaus1 and gaus2 to 0.0 for stress evaluation.

The K matrix for **reduced integration** of Timoshenko Beam is taken as follows,

$$K = \frac{GA^*}{L}\begin{bmatrix} 1 & \frac{L}{2} & -1 & \frac{L}{2} \\ & \frac{L^2}{4} & \frac{-L}{2} & \frac{L^2}{4} \\ & & 1 & \frac{-L}{2} \\ Symm. & & & \frac{L^2}{4} \end{bmatrix}$$

```
const = D_matb/len;

K_b  = [ 0 ,  0 ,  0 ,  0 ;
         0 ,  1 ,  0 , -1 ;
         0 ,  0 ,  0 ,  0 ;
         0 , -1 ,  0 ,  1 ];

K_b  = K_b * const;

const =   D_mats/len;

K_s = [   1    ,   len/2 ,    -1    ,    len/2 ;
            len/2 , len^2/4 , -len/2 , len^2/4 ;
             -1    ,  -len/2 ,     1    ,  -len/2 ;
            len/2 , len^2/4 , -len/2 ,  len^2/4 ];

K_s = K_s * const;
```

```
% One gauss point for stress evaluation
gaus0 =  0.0;     % One Gauss point for stresses evaluation

bmat_b=[ 0, -1/len, 0, 1/len];

bmat_s1=[-1/len,-(1-gaus0)/2, 1/len,-(1+gaus0)/2];

Str1_g0 = D_matb*(bmat_b *transpose(u_elem));
Str2_g0 = D_mats*(bmat_s1*transpose(u_elem));

Strnod(lnods(1),1)  = Strnod(lnods(1),1)+Str1_g0;
Strnod(lnods(2),1)  = Strnod(lnods(2),1)+Str1_g0;
Strnod(lnods(1),2)  = Strnod(lnods(1),2)+Str2_g0;
Strnod(lnods(2),2)  = Strnod(lnods(2),2)+Str2_g0;
Strnod(lnods(1),3)  = Strnod(lnods(1),3)+1;
Strnod(lnods(2),3)  = Strnod(lnods(2),3)+1;
```

b) The problem for 64 element mesh is solved for following cases:
   i) 2 nodes Euler Bernoulli element
   ii) 2 nodes Timoshenko Full Integrate element
   iii) 2 nodes Timoshenko Reduce Integration element.

- The deflection for simple supported Euler Bernoulli beam, is calculated analytically by the following formula,

$$\delta = \frac{5PL^4}{384EI}$$

Whereas, deflection for simple supported Euler Bernoulli beam, is calculated analytically by the following formula,

$$\delta = \frac{5PL^4}{384EI} + \frac{I}{GA^*}$$

where,

$E = 21000$; $\upsilon = 0.25$; $P = 1.0$; L= 4m

$G = 2(1+\upsilon) = 2X(1+0.25) = 2.5$; $I = \frac{b*h^3}{12} = \frac{a^4}{12}$;

$A^* = \frac{5}{6}A = \frac{5}{6}(a*a)$ …. for rectangular/square cross sectional area.

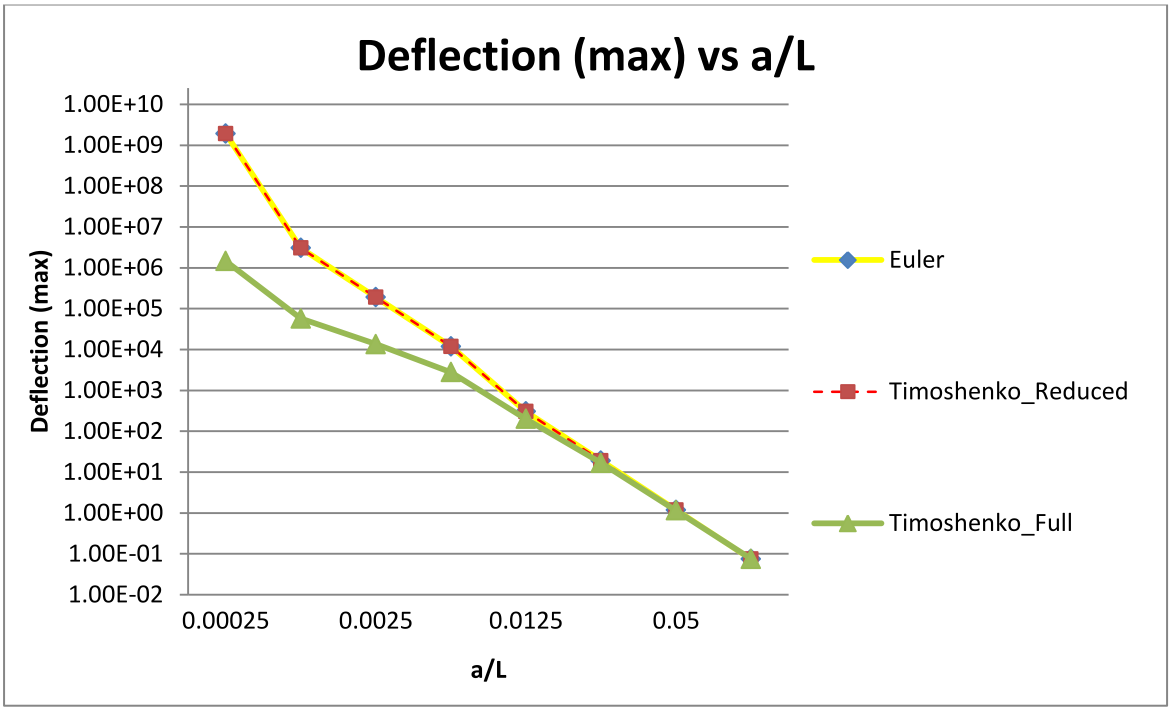

Figure 1:Maximum Deflection vs aspect ratio (a/L) for Euler Bernoulli and Timoshenko beam

The above graph shows the plot of obtained values for maximum deflection by solving the Matlab code for all three cases. The obtained values are very similar to the analytical solution, which verifies that the results have converged.

From the graph, it can be observed that Euler Bernoulli and Timoshenko Reduced order give same values of deflection whereas the Timoshenko full order integration gives slightly deviation in results for lower aspect ratio (a/L). This shows that for thin beams, Timoshenko Reduced order integration method gives accurate results and for thick beams all the three methods give better accuracy of results.

Euler Bernoulli								
	a	a/L	W			Moment		% Error
			actual	obtained		actual	obtained	
	0.001	0.00025	1.90E+09	1.90E+08		2	1.99	-9.00E+01
	0.005	0.00125	3.05E+06	3.05E+06		2	2.003	1.97E-02
	0.01	0.0025	1.90E+05	1.90E+05		2	2.003	1.58E-03
	0.02	0.005	1.19E+04	1.19E+04		2	2.003	3.36E-02
	0.05	0.0125	3.04E+02	3.04E+02		2	2.003	6.58E-02
	0.1	0.025	19.04	19.041		2	2.003	5.25E-03
	0.2	0.05	1.19	1.191		2	2.003	8.40E-02
	0.4	0.1	0.0744	0.07439		2	2.003	-1.34E-02

- The bending moment for simply supported Euler Bernoulli and Timoshenko beam is given by,

$$M = \frac{PL^2}{8} = \frac{1 * 4^2}{8} = 2\ Nm$$

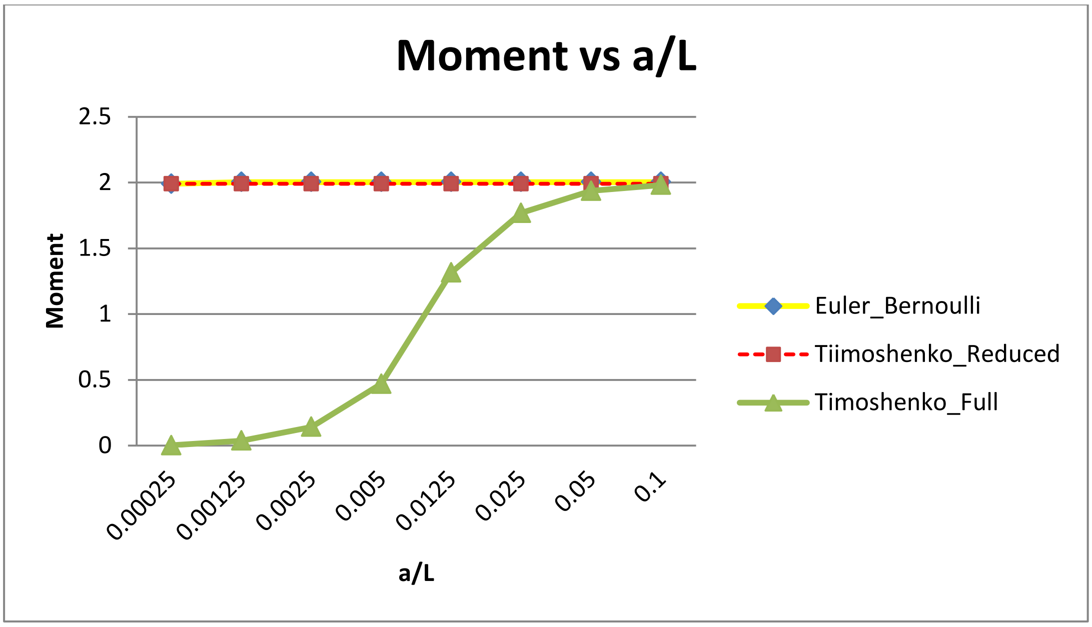

Figure 2: Maximum Moment vs aspect ratio (a/L) for Euler Bernoulli and Timoshenko beam

The bending moment is independent of 'a' and hence should remain constant along a/L. From the above figure it can be observed that Euler Bernoulli and Timoshenko Reduced method give better accuracy of results whereas Timoshenko Full gives bad prediction of results for lower aspect ratio (a/L). Timoshenko Full integration is best suited for thicker beams.

It can be seen that, Euler - Bernoulli Beam elements give good results for normal stress, because they are capable of capturing bending dominated deformation fields. If a beam is not slender and it goes into bending dominated deformation then Timoshenko elements are weak to capture normal stress and Euler - Bernoulli beam elements are weak to capture shear deformation.

Timoshenko reduced integration										
						Moment				
	a	a/L	W			M		Q		%Error W
			actual	obtained		actual	obtained	actual	obtained	
	0.001	0.00025	1.90E+09	1.90E+09		2	1.99	2	1.9688	0.00E+00
	0.005	0.00125	3.05E+06	3.05E+06		2	1.99	2	1.9688	-1.97E-02
	0.01	0.0025	1.90E+05	1.90E+05		2	1.99	2	1.9688	0.00E+00
	0.02	0.005	1.19E+04	1.19E+04		2	1.99	2	1.9688	-4.20E-02
	0.05	0.0125	3.04E+02	3.04E+02		2	1.99	2	1.9688	0.00E+00
	0.1	0.025	19.04	19.048		2	1.99	2	1.9688	-4.20E-02
	0.2	0.05	1.19	1.1905		2	1.99	2	1.9688	-4.20E-02
	0.4	0.1	0.0744	0.074405		2	1.99	2	1.9688	-6.72E-03

- The shear force for simply supported Euler Bernoulli and Timoshenko beam is given by,

$$Q = \frac{PL}{2} = \frac{4}{2} = 2\ N/m$$

In Euler-Bernoulli beams, transverse shear stress is not taken into account whereas in Timoshenko beams transverse shear stresses are taken into account. The reason why transverse shear stress is not taken into account in Euler - Bernoulli beams is bending is assumed to behave in such a way that cross section normal to the neutral axis remain normal to the neutral axis after bending. In case of Timoshenko beams initially cross section in normal to the neutral axis but does not remain normal after bending.

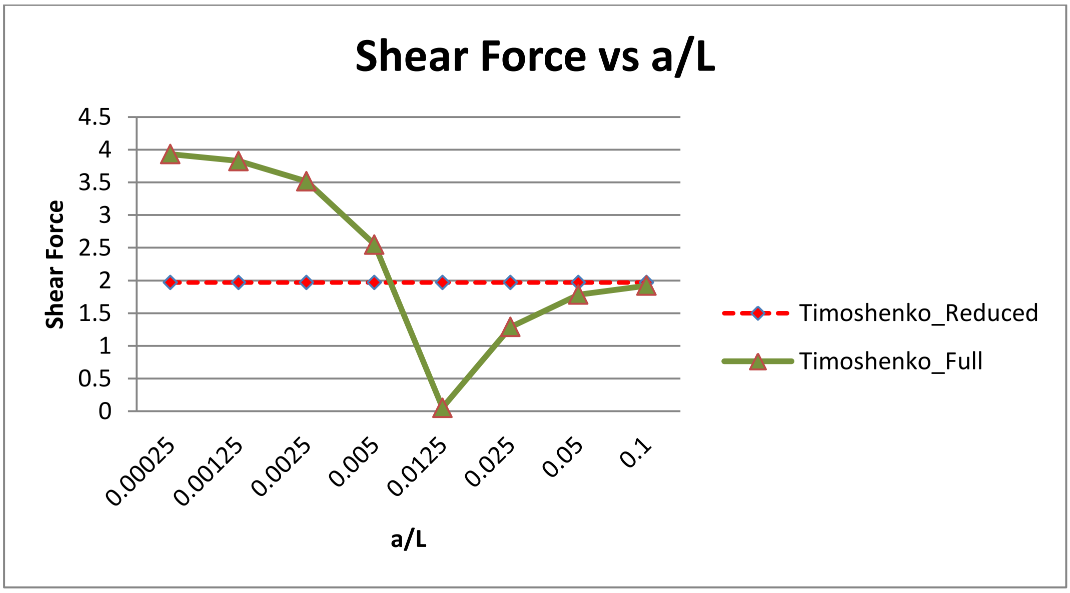

Figure 3 : Maximum Shear Force vs aspect ratio (a/L) for Euler Bernoulli and Timoshenko beam

From the above figure it can be seen that Timoshenko Reduced integration gives accuracy in results for shear force against a/L ratio. The Timoshenko Full integration method doesn't give accuracy of results for lower a/L ratio. Hence, Timoshenko Full integration can be used for thick beams. For thin beams, Timoshenko Reduced integration method is more preferable.

Timoshenko Full Integration									
a	a/L	W			M		Q		
					actual	Obtained	actual	Obtained	% Error W
0.001	0.00025	1.90E+09	1.46E+06		2	0.00153	2	3.933	-9.99E+01
0.005	0.00125	3.05E+06	5.74E+04		2	0.0376	2	3.8262	-9.81E+01
0.01	0.0025	1.90E+05	1.36E+04		2	0.1425	2	3.5162	-9.29E+01
0.02	0.005	1.19E+04	2.80E+03		2	0.4697	2	2.5495	-7.65E+01
0.05	0.0125	3.04E+02	2.00E+02		2	1.3144	2	0.0539	-3.41E+01
0.1	0.025	19.04	1.69E+01		2	1.7687	2	1.2883	-1.14E+01
0.2	0.05	1.19	1.16E+00		2	1.936	2	1.7825	-2.55E+00
0.4	0.1	0.0744	7.55E-02		2	1.982	2	1.921	1.48E+00