

# Computational Structural Mechanics and Dynamics -2017

Practice # 1

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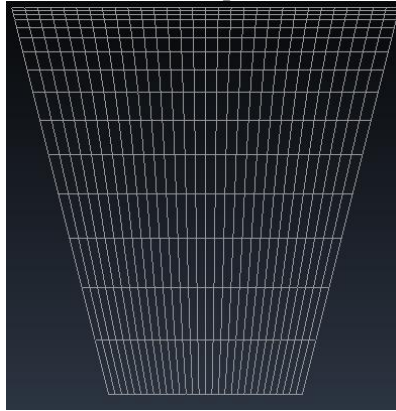
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**Exercise 1:** the quadrilateral and triangular meshes employed are shown below

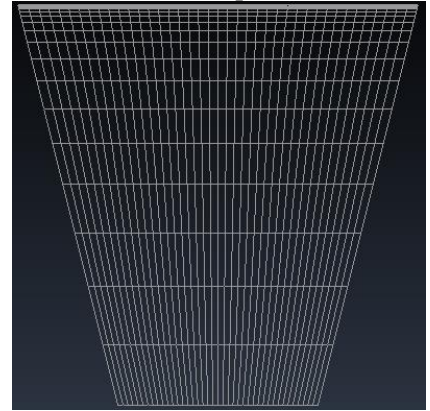
Unrefined quadrilateral



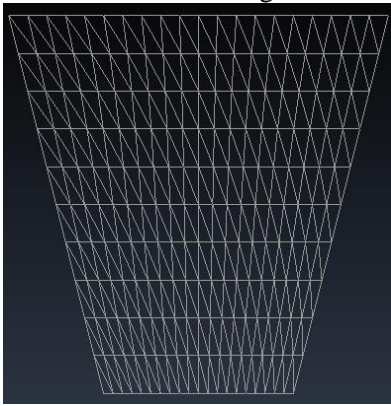
Refinement 1 quadrilateral



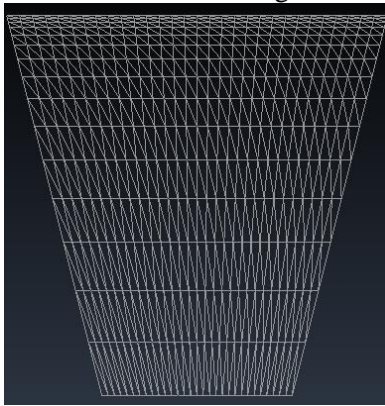
Refinement 2 quadrilateral



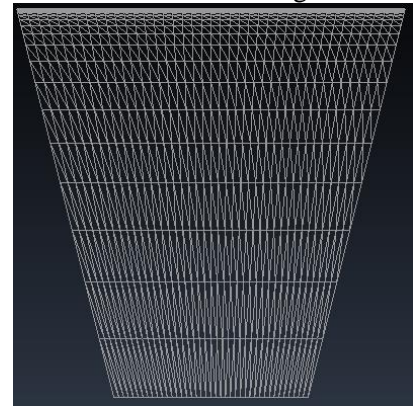
Unrefined triangular



Refinement 1 triangular



Refinement 2 triangular



Results: The vertical displacement of the point midway along segment ED and the Y-component of stress at point B are set out in the table below. The columns refer to mesh refinements (Unrefined, Refinement 1 and Refinement 2) and the rows to the number of nodes and type of element (3 and 6 node triangular and 4, 8 and 9 node quadrilateral elements).

	Vertical displacement $U_Y$ and stress $\sigma_Y$					
	$U_Y$ at ED midpoint ( $\times 10^{-6}$ m)			$\sigma_Y$ at B ( $\times 10^5$ MN/m <sup>2</sup> )		
	Unref.	Ref. 1	Ref. 2	Unref.	Ref. 1	Ref. 2
<b>Tri. 3</b>	-2,30	-2,31	-2,31	0,231	0,250	0,251
<b>Tri. 6</b>	-2,30	-2,30	-2,30	0,247	0,251	0,252
<b>Quad. 4</b>	-2,60	-2,31	-2,31	0,133	0,225	0,252
<b>Quad. 8</b>	-2,30	-2,30	-2,30	0,249	0,252	0,252
<b>Quad. 9</b>	-2,30	-2,30	-2,30	0,249	0,252	0,252

The relative errors (computed as the absolute value of the difference between calculated and reference values divided into the reference value) are shown next:

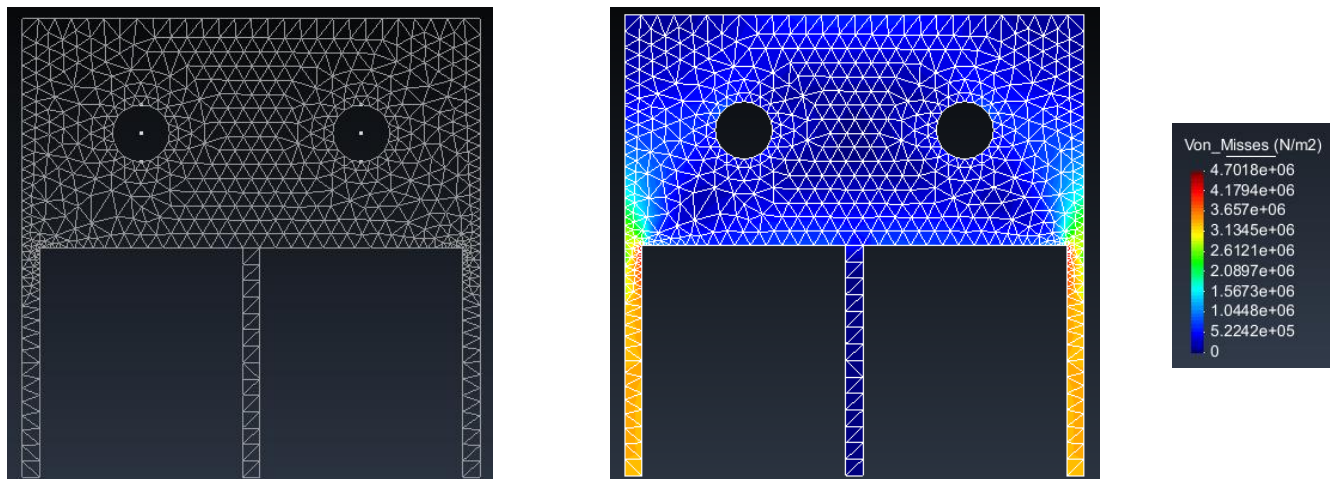
	Relative Error (%)					
	$U_Y$ at ED midpoint ( $\times 10^{-6}$ m)			$\sigma_Y$ at B ( $\times 10^5$ MN/m <sup>2</sup> )		
	Unref.	Ref. 1	Ref. 2	Unref.	Ref. 1	Ref. 2
<b>Tri. 3</b>	1,9%	2,0%	2,0%	6,5%	1,2%	1,6%
<b>Tri. 6</b>	1,9%	1,9%	1,9%	0,0%	1,6%	2,0%
<b>Quad. 4</b>	14,9%	2,0%	2,1%	46,2%	8,9%	2,0%
<b>Quad. 8</b>	1,9%	1,9%	1,9%	0,8%	2,0%	2,0%
<b>Quad. 9</b>	1,9%	1,9%	1,9%	0,8%	2,0%	2,0%

When using triangular elements for computing displacement, there is not much improvement either by mesh refinement or by raising the order of the element. It was the case, however, when going from low to high order quadrilaterals, and particularly so when refining the mesh for the bilinear quadrilateral.

Roughly the same conclusions can be drawn for the calculation of the stress component. Nevertheless, in this case the improvement due to mesh refinement using triangular elements is more evident.

Mesh refinement was applied to the top end of the plate due to the foreseeable higher gradients of displacement.

**Exercise 2:** The mesh included point refinement on the leftmost and rightmost entrant corners of the plate. It is shown on the left of the figure below. The distribution of the Von Mises stress is shown on the right.



The static equilibrium of the plate requires only two supporting columns, so that removing the middle column footing causes the side columns to take up the whole of the load. Additionally, the relative size of the plate with respect to the side columns concentrates the stress at the entrant corners corresponding to change of sections.

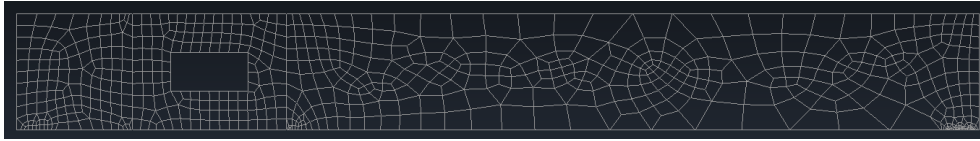
**Exercise 3:** Here we are going to substitute the reinforcing effect of the steel plates by raising the width of the concrete beam, so as to get an equivalent cross section according to the ratio of their elastic moduli:

$$\frac{E_{steel}}{E_{concrete}} = \frac{3.0}{2.1} = 1.43$$

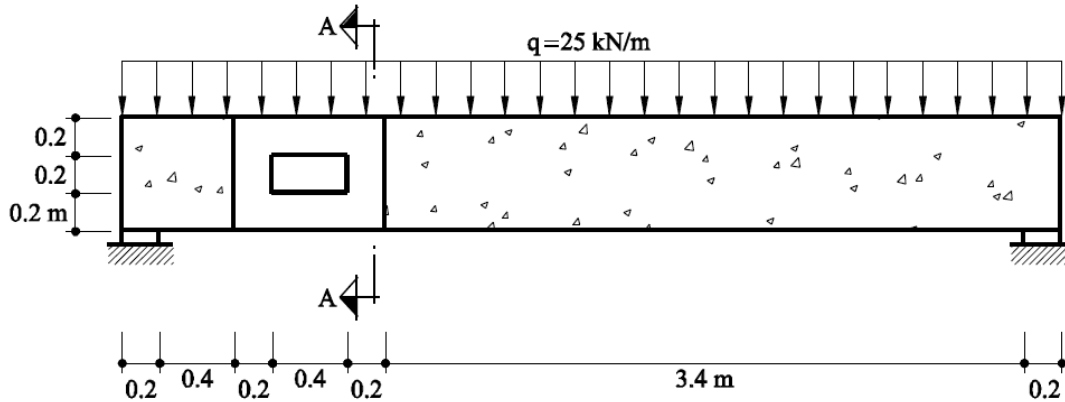
Then, for this problem, the equivalent thickness of a beam made up of concrete alone is:

$$t_{eq} = (1.43 \cdot 0.008m \cdot 2) + 0.25m = 0.27288m$$

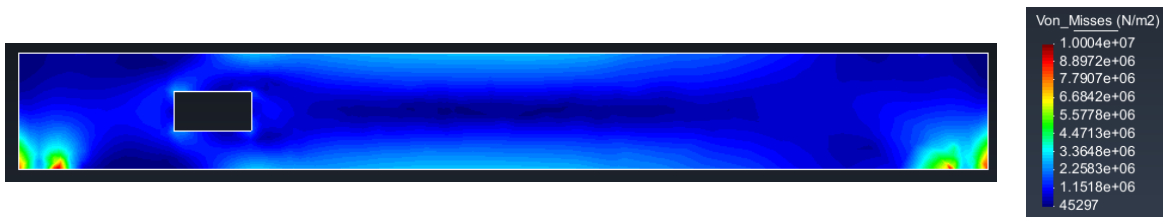
The mesh employed is shown below:



The boundary conditions were Y-constrained motion at the support/beam interfaces and X-Y-constrained motion at the bottom left corner of the beam, to prevent rigid body motions.

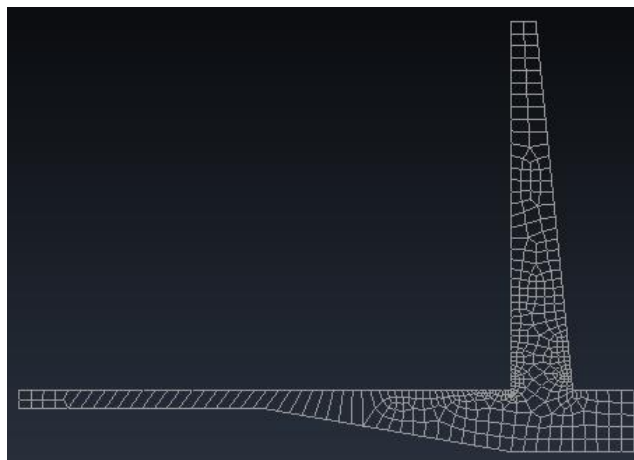


The Von Mises stress distribution is shown next:

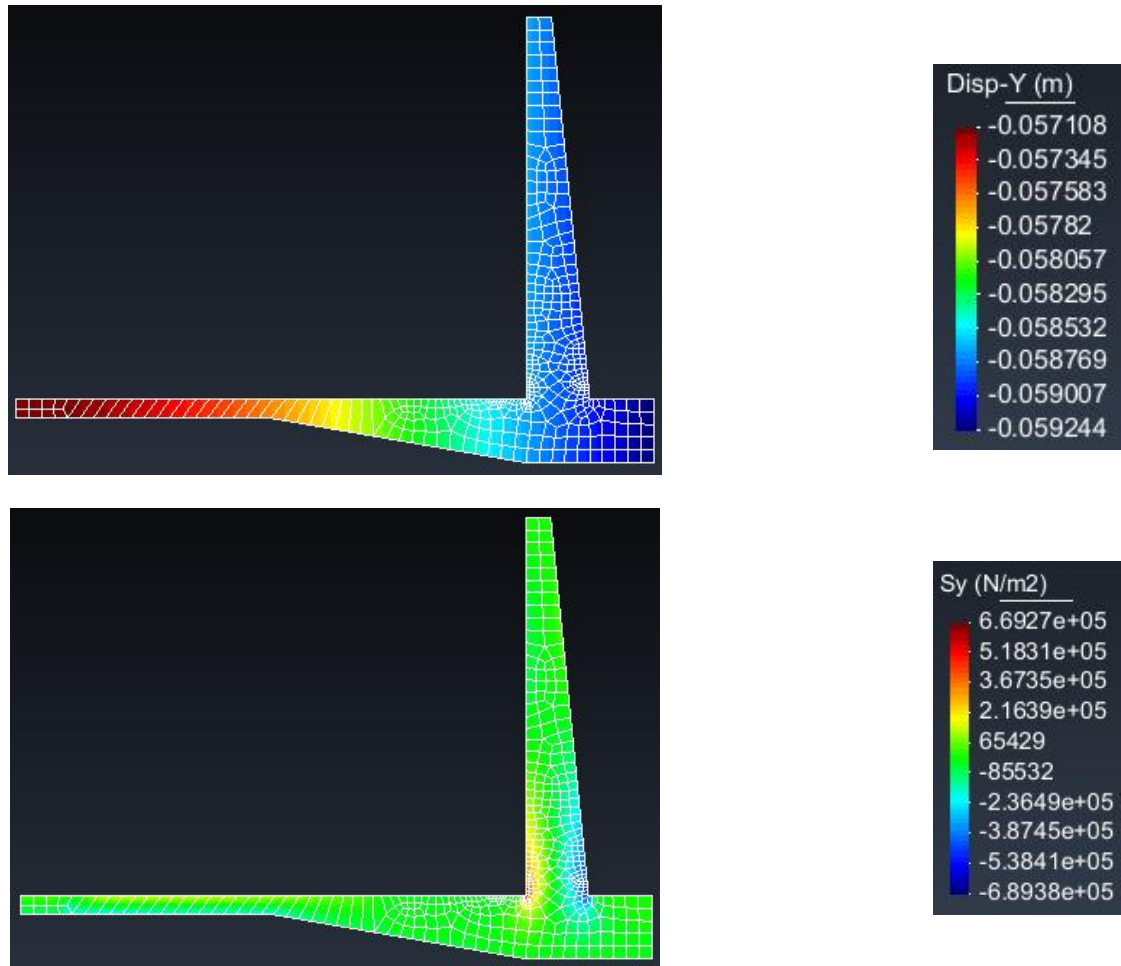


The reinforcing steel plate is equivalent to an increased load bearing section made up of concrete. This allows reducing stress in zones otherwise critical, such as that between the left support and the ventilation hole. Stress concentration is nonetheless present at localised regions about the end of each support.

**Exercise 4:** The mesh employed is shown next:



The Y-stress and Y-displacement distribution seem relevant in this analysis and are therefore shown below:



The displacements show small variations within the domain, but allow telling that the elastic support from the ground contributes the most to this result.

The maximum stresses occur about the stress concentration on the base of the wall; they are tensile on the left and compressive on the right side of it. Comparatively, the compression stress due to the weight of the water on the tank bottom is small.