Computational Structural Mechanics and Dynamics

GID Practice 2
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Exercise 1:

1. The figure shows a circular tank made of reinforced concrete. It is used for the storage of water in a water purification plant. Analyze the structural behavior of the tank. Use quadrilateral elements with four nodes.

   Material properties

$$E_s = 3.010^{11}\frac{N}{m^2} \tag{1}$$

$$\nu_s = 0.2 \tag{2}$$

$$Floor - Ballastercoef = 50\frac{N}{cm^3} \tag{3}$$

$$\tag{4}$$

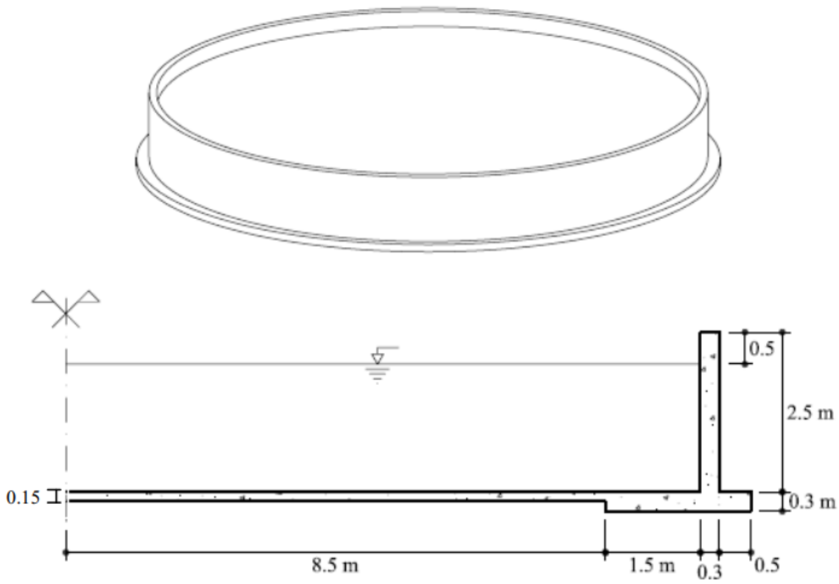

Figure 1: geometry

2. It was considered a revolution solid. The FEM model was meshed with a total of 1481 quadrilateral linear elements. Axilsymmetric Constraints were imposed. Elastic constraints were

imposed in order to simulate the reaction of the floor. Uniform load was applied on the floor of the tank and a linear distribution load was applied on the interior wall of the tank to simulation the water loading.

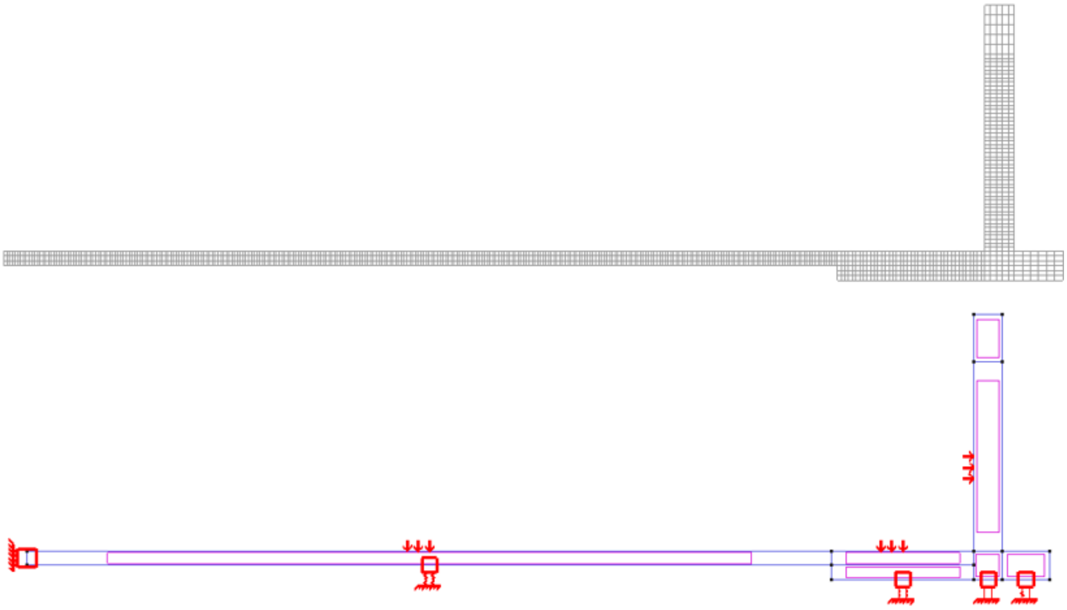

Figure 2: Mesh and Boundary conditions.

Figure 3: Total displacements [mm]

The total displacements are in accordance with loads and constraints applied.

Figure 4: σ_V Von Misses stresses [MPa]

It can be seen the max value of Von Misses stresses was less than yield stress. So, the structure showed a linear elastic behaviour.

Exercise 2:

1. The exercise asks to compute the deformed state of a fixed cantilever under the action of a certain moment acting on its free end, and then compare the results with those obtained with beam theory.

   Material properties

$$E_s = 2.110^{11} \frac{N}{m^2} \tag{5}$$
$$\nu_s = 0.2 \tag{6}$$

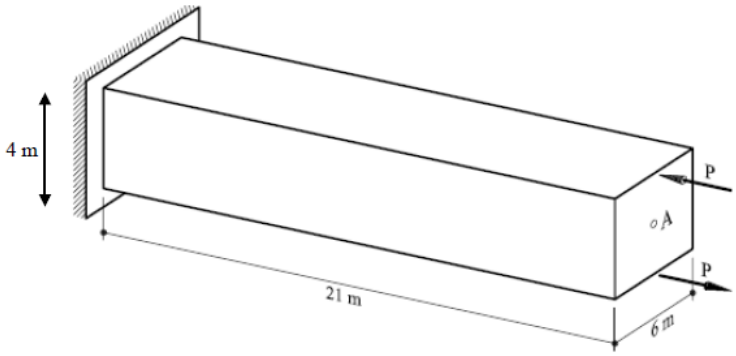

Figure 5: Problem geometry

2. The FEM model was meshed with a total of 2100 elements. For linear hexaedra, we have a total of 2662 nodes. For quadratic 20-noded hexaedra, we have a total of 10043 nodes. Constraints were imposed so displacements $u_x = u_y = u_z = 0$ for those points with X = 0.

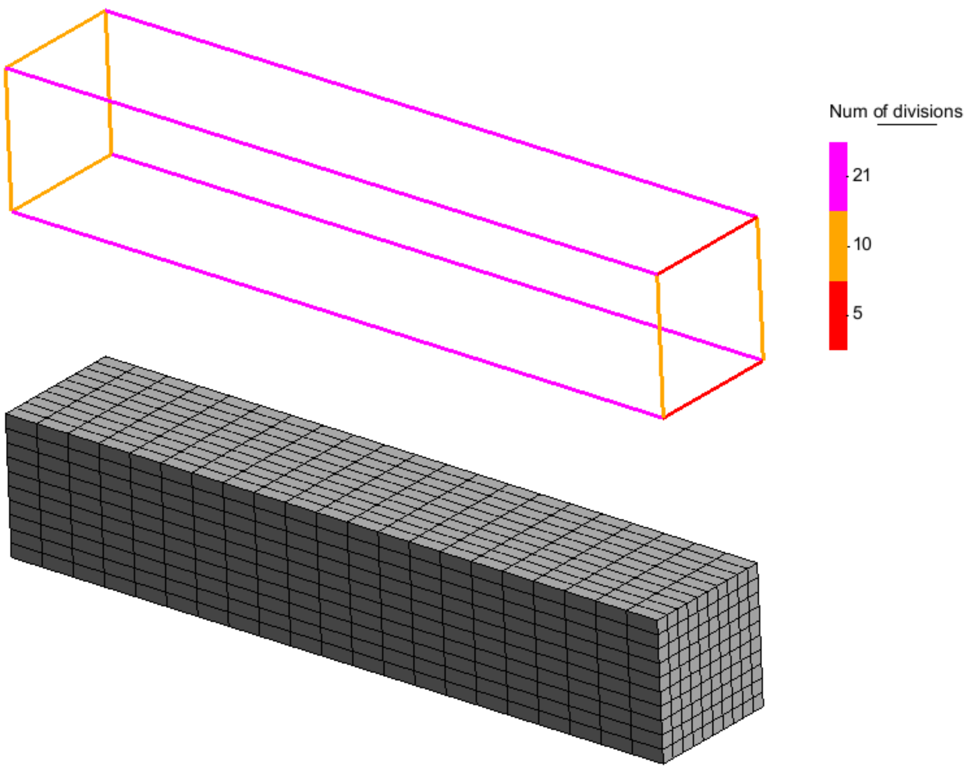

Figure 6: Mesh divisions and mesh. The model has 2100 elements

3. In order to compare the results, we have to indicate that following beam theory the two forces that appear in the scheme (see figure 5) can be considered as if they were inducing only a constant moment (no axial or tangential forces) into the beam. This will create a positive vertical bending of the beam, and a constant stress σ_x over the section. However, our FEM model is more complete, using to opposed forces, and accounting for Poisson effect and the three dimensional geometry of the real element, meaning that the computer-based results will include displacements in the horizontal plane and stresses in y and z direction, which we will not consider. Moreover, these stresses won't be the same throughout the whole beam, but concentrated in the application points of the force, as classical beam theory accounts for forces acting all over the section (distributed loads) and not point loads.

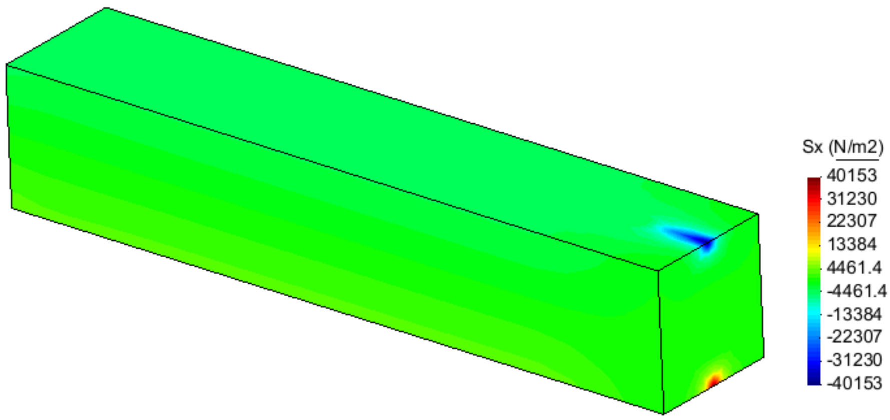

Figure 7: σ_x stresses for the 8 node hexaedra model

4. For the mesh of 8 nodes hexaedra, maximum deflection at middle point (axis of the beam) and stresses are

$$u_z = 1.9110^{-6}m \tag{7}$$
$$\sigma_x = 40000\frac{N}{m^2} \tag{8}$$

5. For the 20 nodes hexaedra, these values are

$$u_z = 1.9710^{-6}m \tag{9}$$
$$\sigma_x = 14905\frac{N}{m^2} \tag{10}$$

6. Following beam theory, the equation governing the deflection of the end point of a cantilever subjected to the action of a moment depends on M, L, E and I_y, which are the moment acting on the end, the beam length, the Young Modulus of the material and the moment of inertia, respectively. In this case, we have a pair of 10 kN over a section of 6x4 creating a moment of $M = 40000Nm$ $21m$ of length, 2.1 $10^{11}\frac{N}{m^2}$ of E and an inertia of $I_y = \frac{64^3}{12} = 32m^4$. Maximum deflection is

$$f_{max} = \frac{ML}{2EI_y} = 6.25\ 10^{-8}m \tag{11}$$

And the maximum stress σ_x is located over the bottommost and the topmost horizontal lines of each section. For rectangular sections this depends on the moment, the section's inertia and the height of those lines (y in the equation). In this case

$$\sigma_{max} = \frac{M}{I_y} y_{max} = \frac{40000}{32} 2 = 2500 Nm \tag{12}$$

We can see that both the maximum displacements and the stresses are lower than in the FEM solution due to the fact that loads are distributed in comparison to the more simple beam theory. Not only that, the values for stresses with linear hexaedra are far from the values for quadratic elements, showing that although the linear interpolation is good enough for displacements, it cannot represent in an adequate form the stresses.

Exercise 3: Foundation of a corner column

1. The problem consist in a corner column with its foundation. This type of foundation is characterized by the fact that the support reaction are eccentric with respect to the load of the column. This results in a flexion of the column and lifting of the base slab. The geometry of the problem is shown in the figure 8.

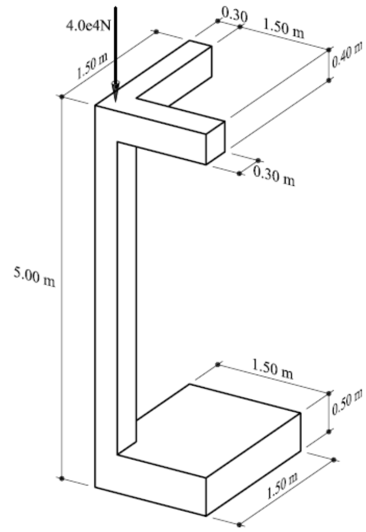

Figure 8: Problem geometry

Concrete material properties

$$E_c = 3.0 \; 10^{10} \; \frac{N}{m^2} \tag{13}$$

$$\nu_c = 0.2 \tag{14}$$

The ballast coefficient of the ground results

$$50 \; \frac{N}{cm^3} \tag{15}$$

Is compared the results in two cases: the first one neglecting the soil over the slab and the second considering $1,5 \; m$ of soil over the base slab. So the ground density considered is

$$\rho = 1800 \; \frac{kg}{m^3} \tag{16}$$

2. The geometry was meshed using structured hexahedrons with eight nodes. In order to ensure a good approach of the solution, and to keep a good aspect ratio, we used a total of 18005 nodes and 14544 elements (0.05 m sided).

3. The results obtained for the first case are shown in the next figures.

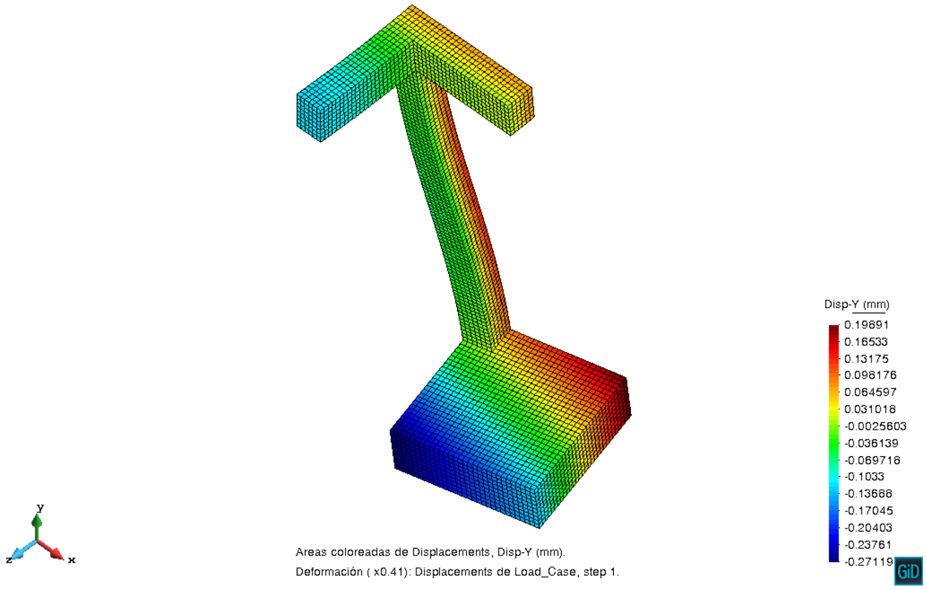

Figure 9: Disp-Y over deformed mesh [mm]

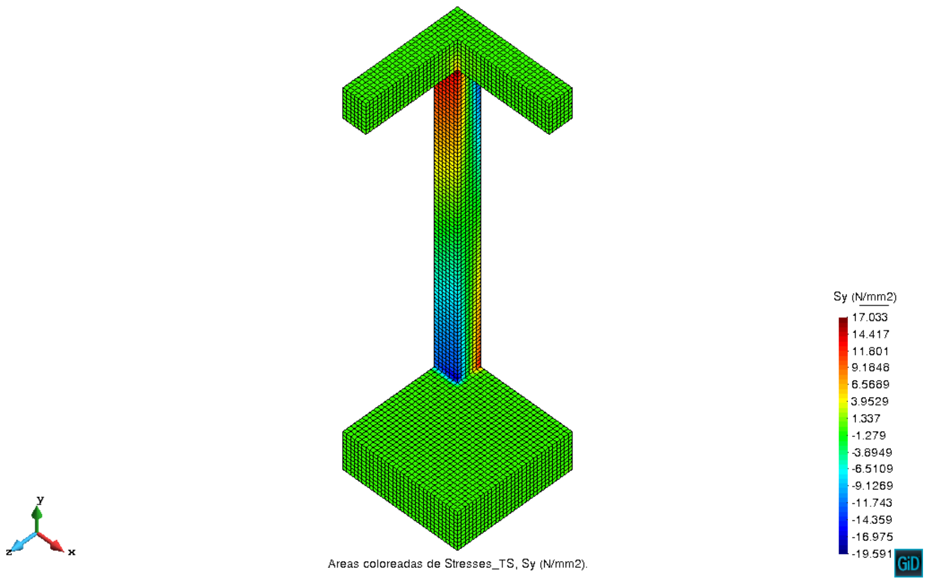

Figure 10: σ_y in column [MPa]

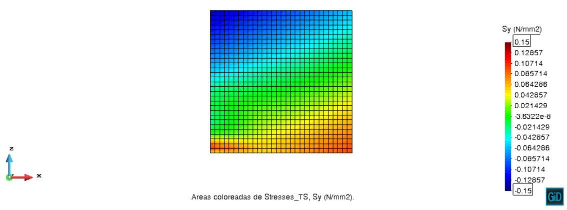

Figure 11: σ_y in slab [MPa]

4. The 9 shows the base slab is lifting in the red corner. Thats occours product of the eccentricity of the load. The tension in the column, see 10 shows the moment it is sufferring with the sides changes of tension and compression. In the top of the column the tension face is the inside, and in the bottom is the outside. So, along the column, the moment changes his sign.

5. The results obtained for the second case are shown in the next figures.

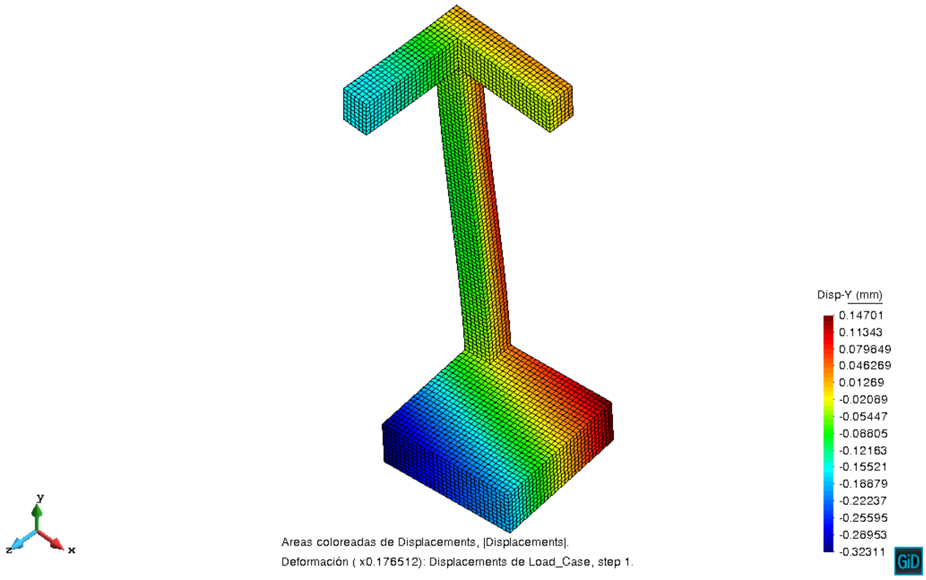

Figure 12: Disp-Y over deformed mesh [mm]

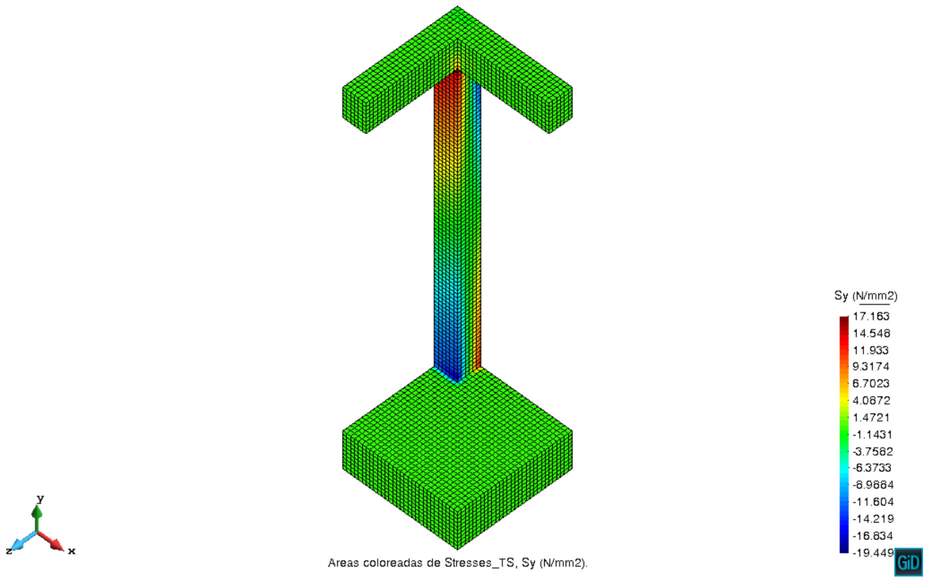

Figure 13: σ_y in column [MPa]

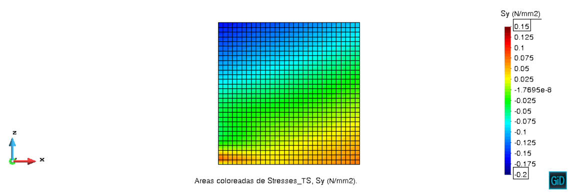

Figure 14: σ_y in slab [MPa]

6. It is observed in the figure 11 that the lifting of the base is reduced but that even considering the weight of the soil, it continues to be produced. There also remains bending in the column.