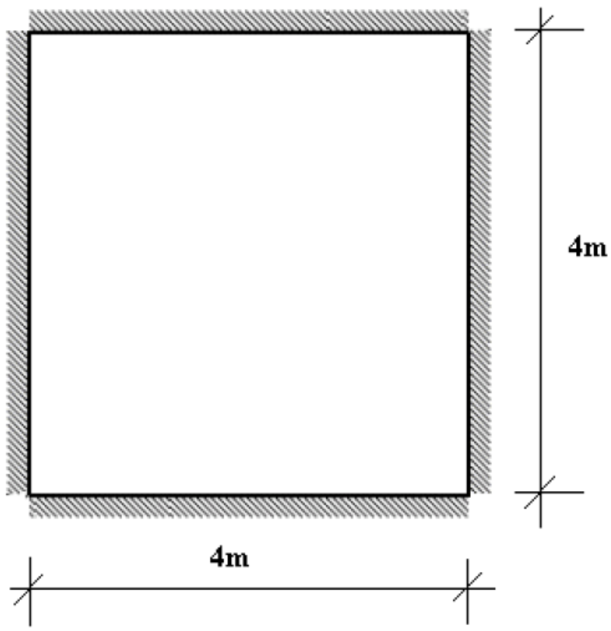
Computational Structural Mechanics and Dynamics

Practice 3

Jose Andino Saint Antonin

Exercise 1: Clamped plate with a uniform load

Analyze the state of stress of the quadratic plate in the figure, whose four sides are clamped. The plate is submitted to a uniform load q. Use triangular plate elements DKT, triangular Reissner-Midlin elements with 6 nodes with reduced integration and quadrilateral elements CLLL for the analysis. Compare the obtained results for the deflection in the center of the plate with the analytical solution.



Data

Concrete
$$\begin{cases} E = 3.0e10 \ \dfrac{N}{m^2} \\ \nu = 0.2 \\ t = 0.10m \text{ (thickness)} \end{cases}$$

Load
$$\left\{ q = 1.0e4 \dfrac{N}{m^2} \right.$$

Solution

Grid:
Since the problem is a square, we can used high quality structured meshes. Splitting the 4m segments on each side into 40 elements each, we get the two following grids for squares and triangles:

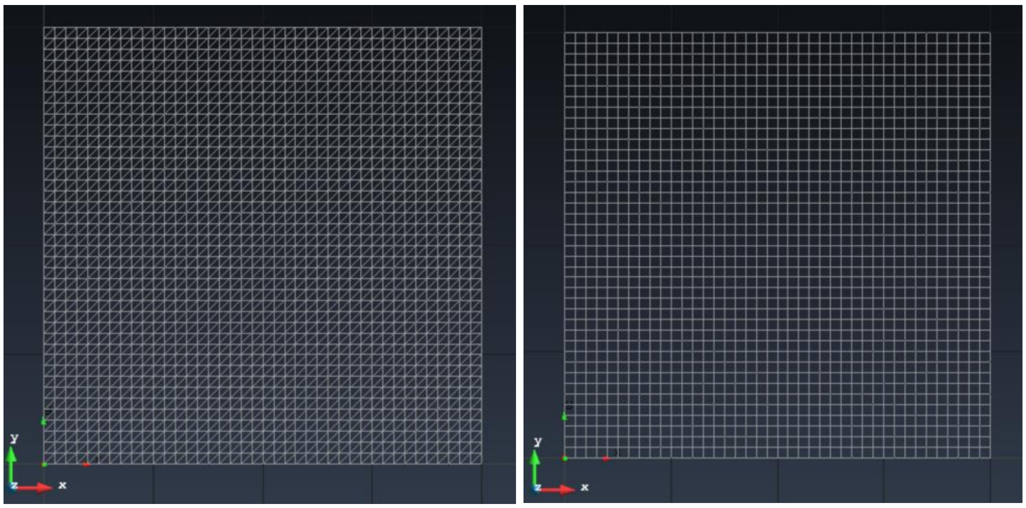


Figure 1 two meshes: on the left using structured triangular elements, on the right using quads.

Notice that the triangular mesh has double the number of elements but same number of nodes:

- Quads: Num. of Quadrilateral elements=1.600, Num. of nodes=1.681
- Triangles: Num. of Triangle elements=3.200, Num. of nodes=1.681

An analysis of element angles is not needed as they are all the same.

Boundary conditions, material properties:
As per the assignment, the edges of the square are prescribed with zero displacement in all directions

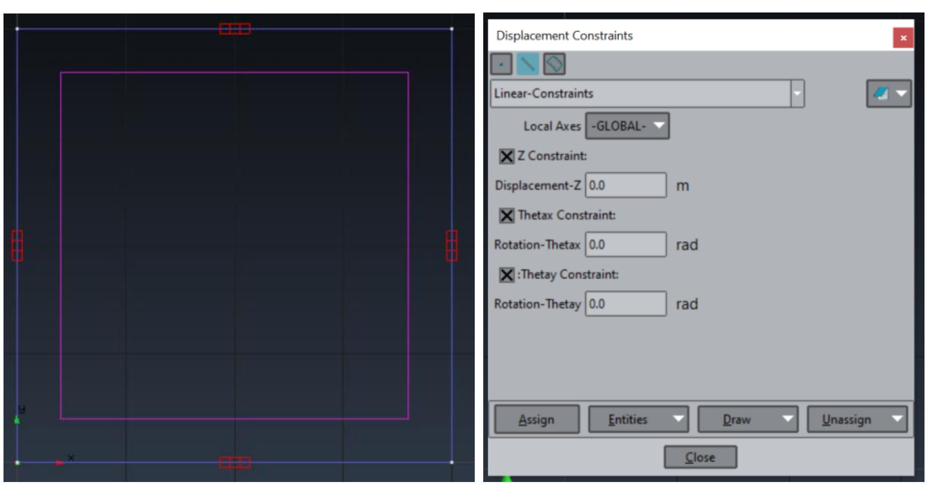


Figure 2 zero displacement prescribed on each of the edges of the plate

The plate is submitted to a uniform load *q in the -z direction*.

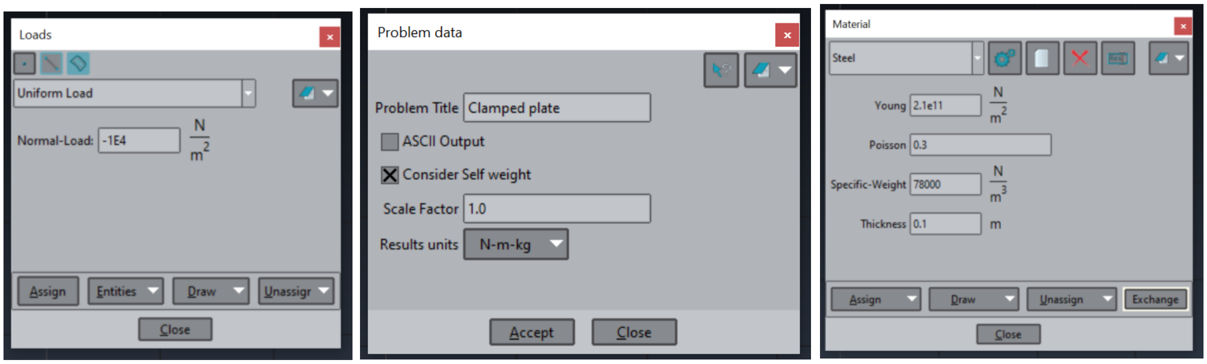
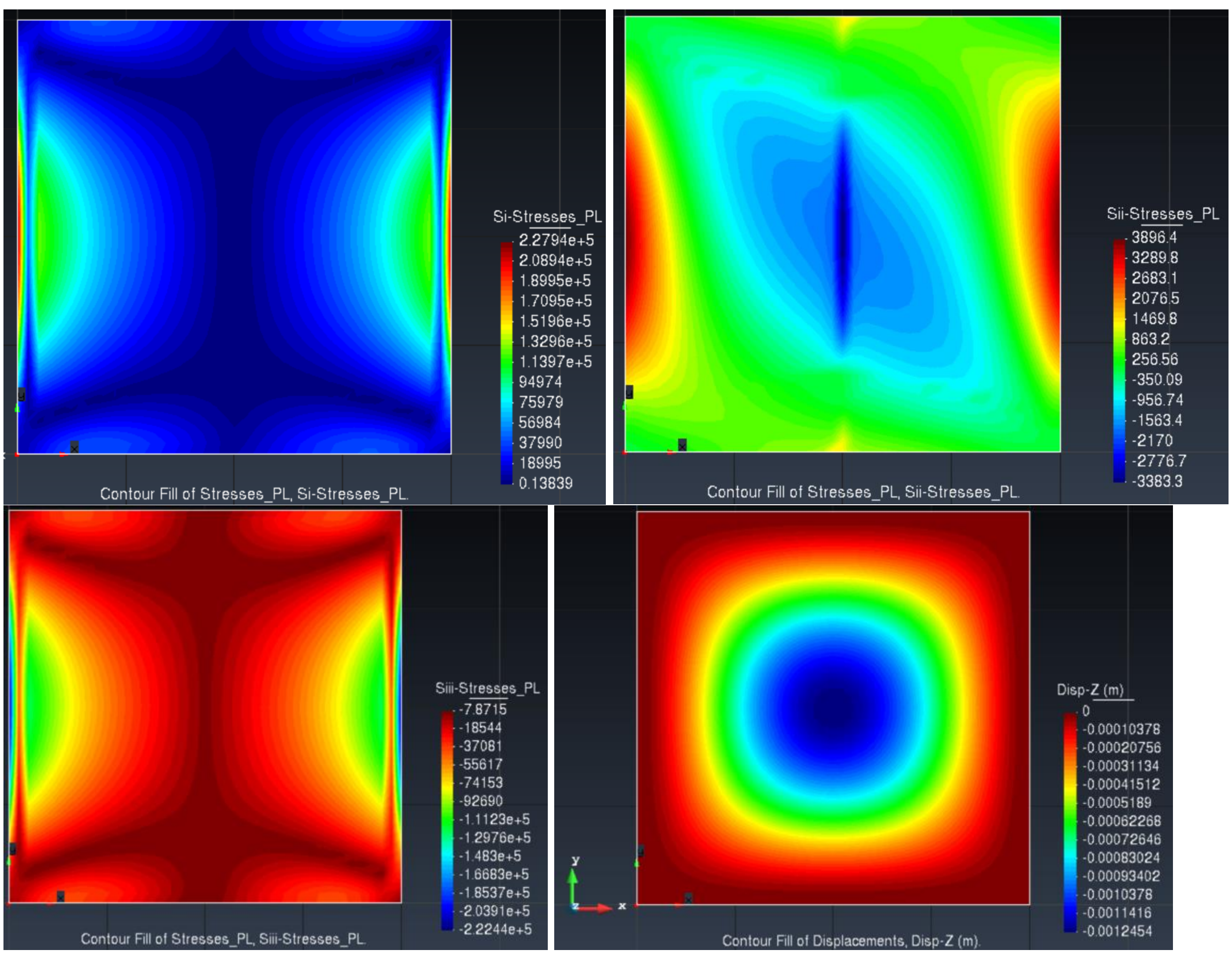
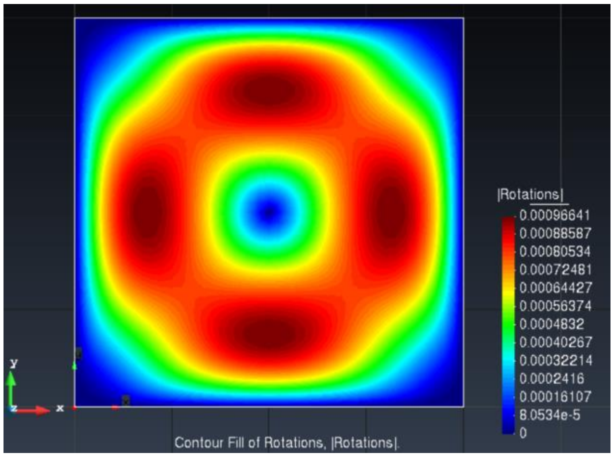


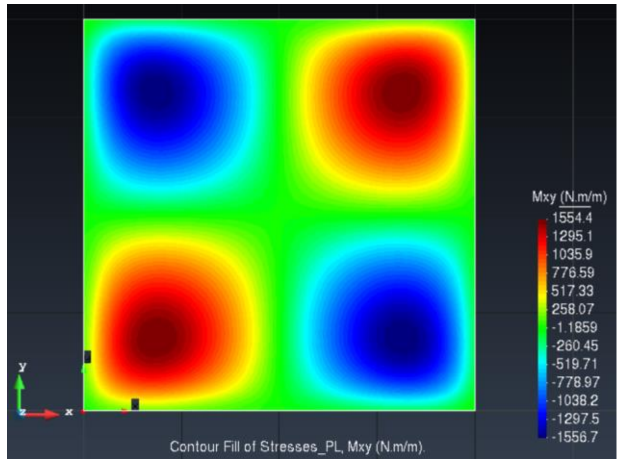
Figure 3 Uniform load, problem type (plus self-weight) and material properties used

Results

DKT results on the triangular meh are shown below. Stresses and vertical displacement are illustrated. As expected, the maximum vertical displacement occurs in the center and the problem is symmetrical.

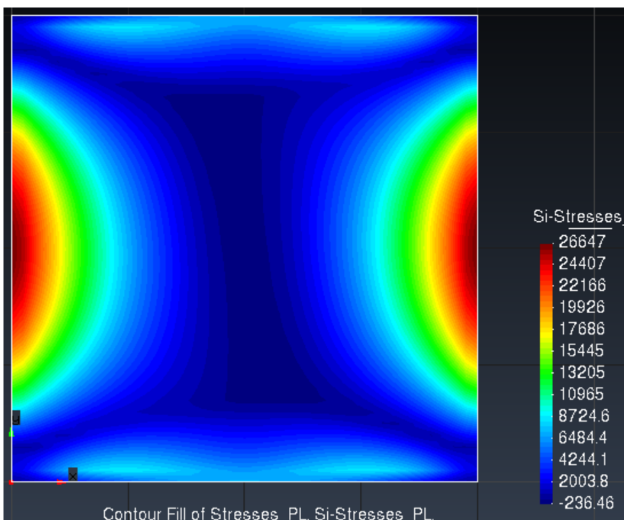


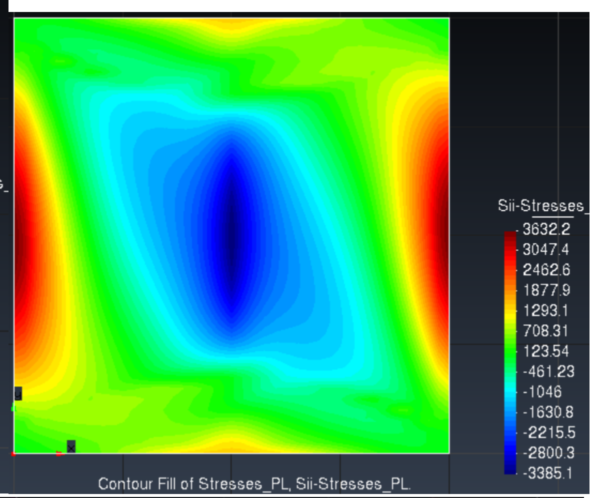


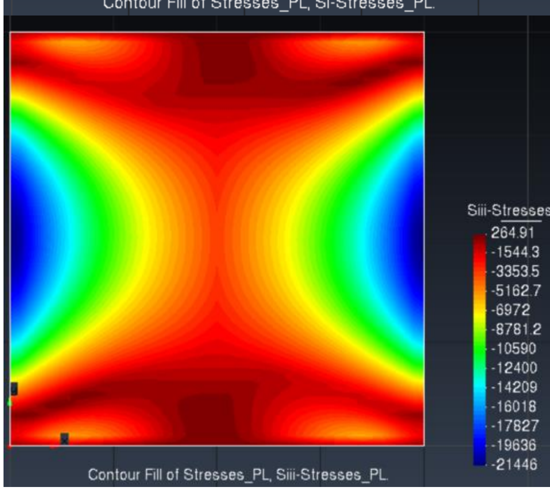


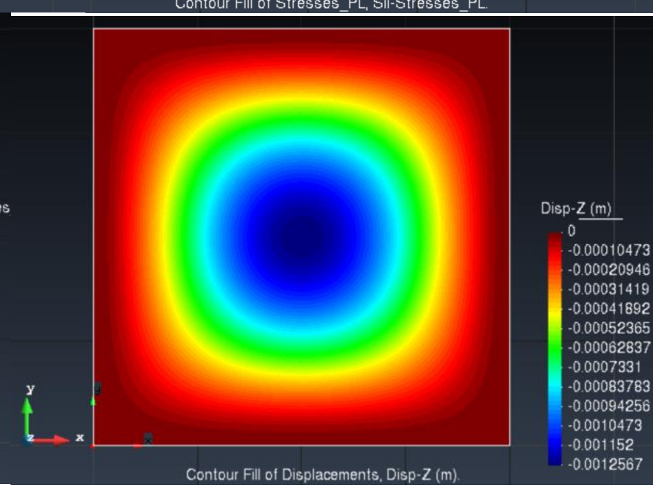
Quads grid CLLL

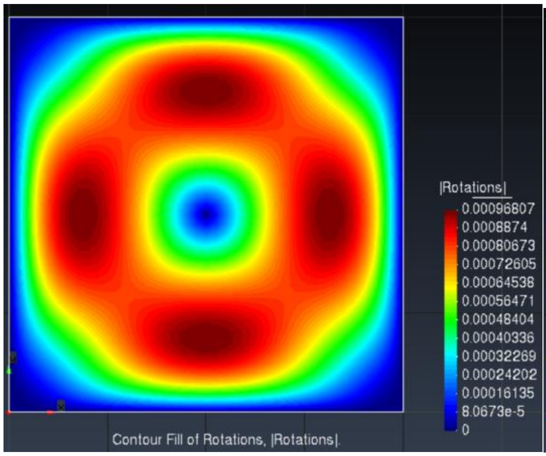
Here are the same results, now shown for the CLLL qudas. Since the grid is quite fine, the results look almost identical

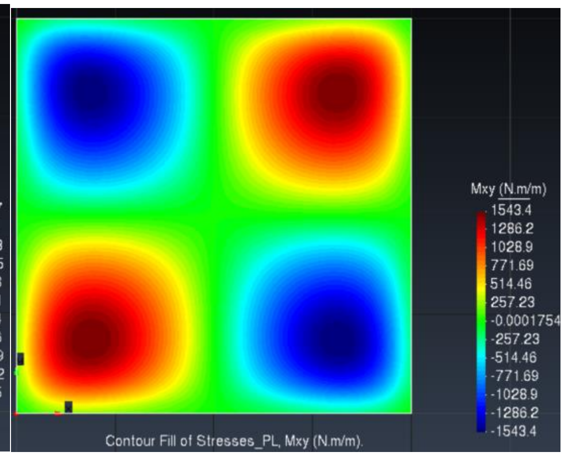












Grid size and node type sensitivity:

As can be observed in the figure & table below (showing vertical displacement for different grid sizes) all three types of elements deliver the same quality of results provided the grid is fine enough (for this particular problem, above 1000 elements they are all in the 1% relative error range). However, CLL and DKT have a clear advantage and are much closer to the 'correct' solution at much lower number of elements. for instance, DKT is closer to the correct vertical displacement with less than 81 nodes than RM is with 289.

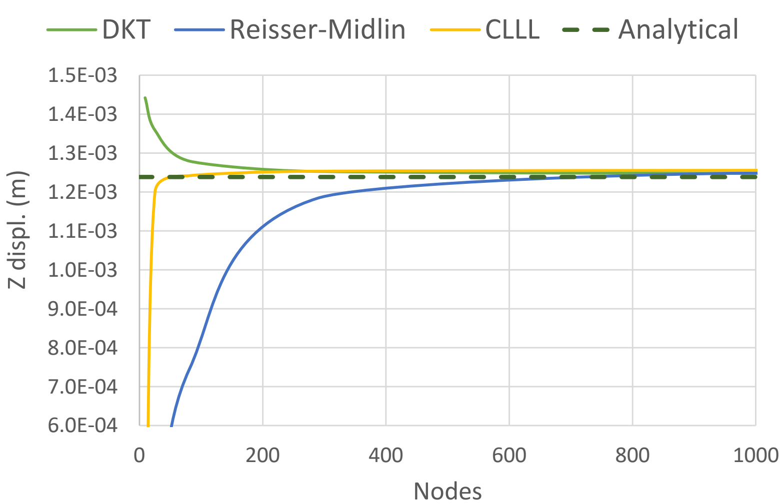


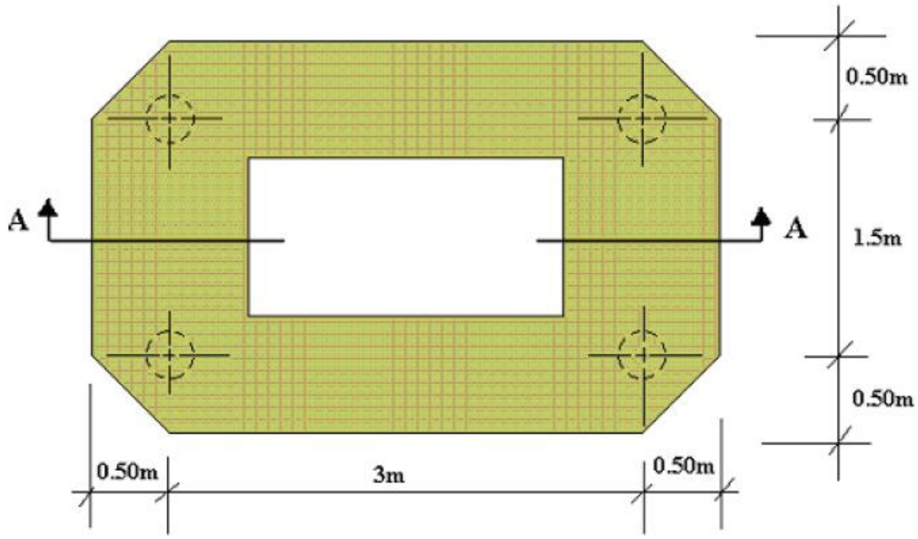
Figure 4 vertical displacement in the center of the plate, vs number of nodes and element type

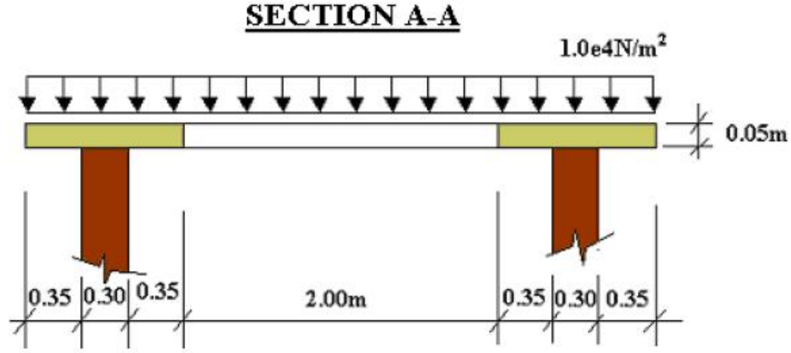
Nodes	DKT	RM	CLL
25	10%	-77%	-3%
81	3%	-40%	0%
289	1%	-4%	1%
1681	1%	1%	1%

Table 1 relative error wrt the analytical solution in vertical displacement at the center of plate

Exercise 2: Thin plate with internal hole

The figure shows a steel plate supported on four columns. Analyze the structural behavior of the plate using the theory of thin plates. Use triangular elements DKT.





Data

$$\text{Steel}\begin{cases} E = 2.1e11\,\dfrac{N}{m^2} \\ \nu = 0.3 \\ \gamma = 7.80e4\,\dfrac{N}{m^3} \end{cases}$$

Solution

Grid

An unstructured was used to model the plate with a hole. The grid angles vary between 28 and 60 degrees which is acceptable.

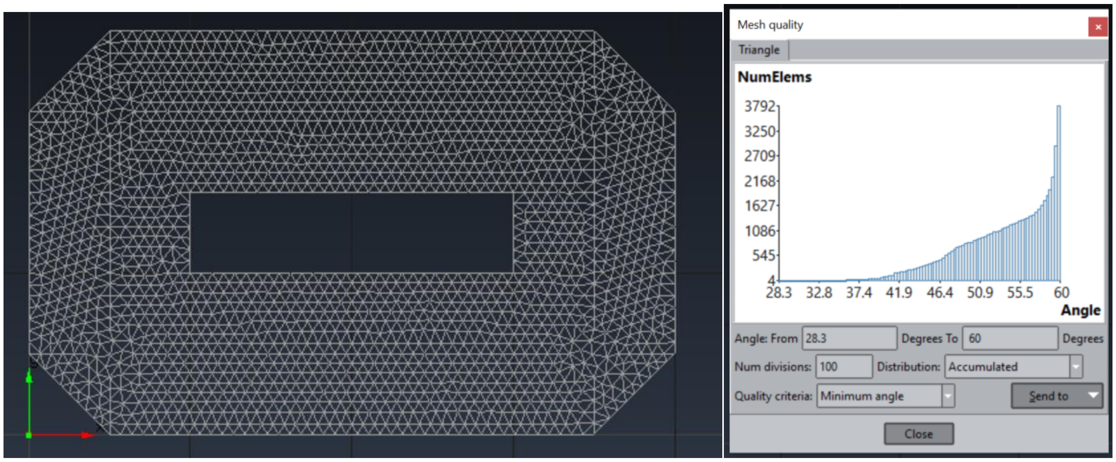


Figure 5 Num. of Triangle elements=3.792, Num. of nodes=2.016

This place theory was used (Ramsiries 2D). Material properties and the uniform load was applied, as shown below.

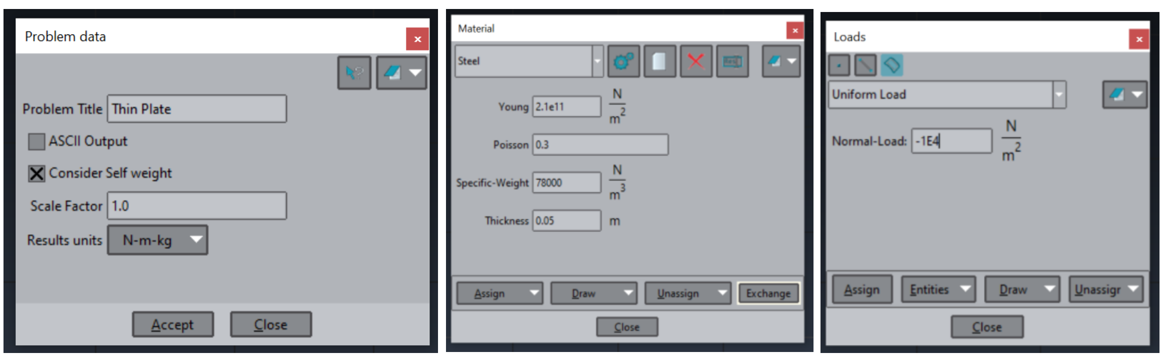


Figure 6 problem type (thin plate with self weight), material properties of steel used, uniform load of -1E4 (z direction)

The no displacement constraint constraint was applied on all the elements that rest on the columns.

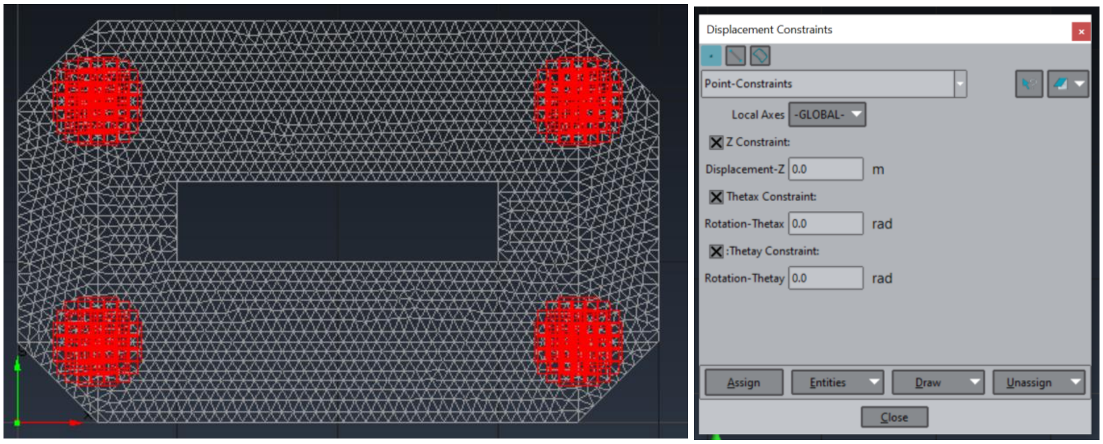


Figure 7   zero displacement constraints in all directions in the red area

Results

As can be seen in the following figure, maximum displacement occurs in the x axis of symmetry.

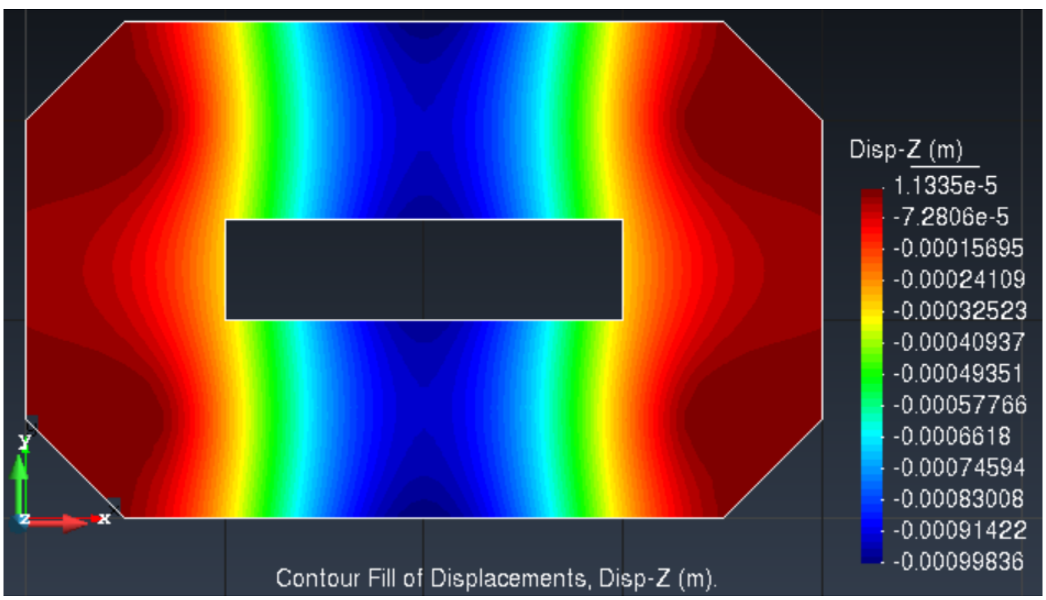


Figure 8   vertical displacements

Stresses are maximum exactly on top of the columns. Not only does this are experience vertical stress (as it is holding the weight of the whole plate) but also tension

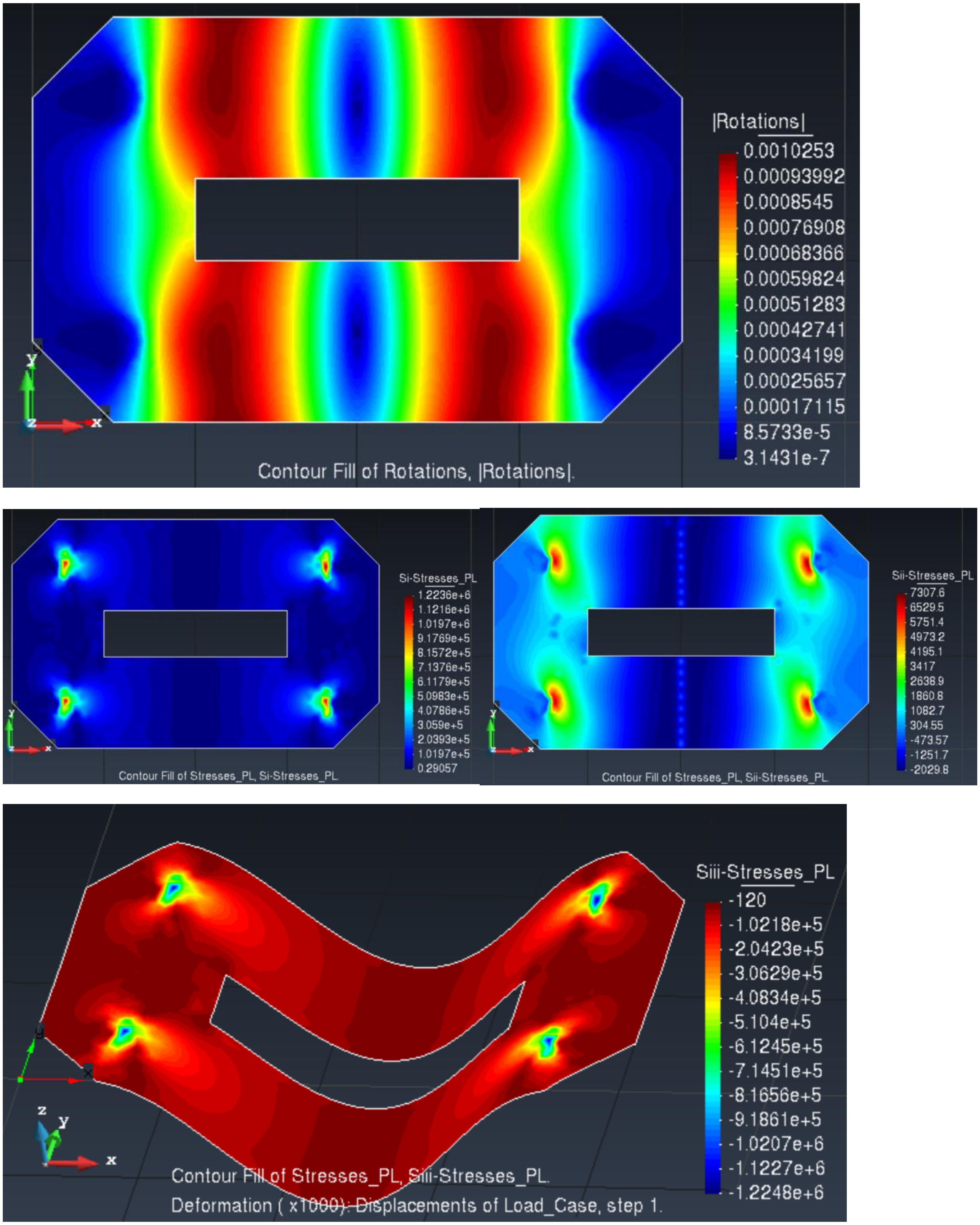
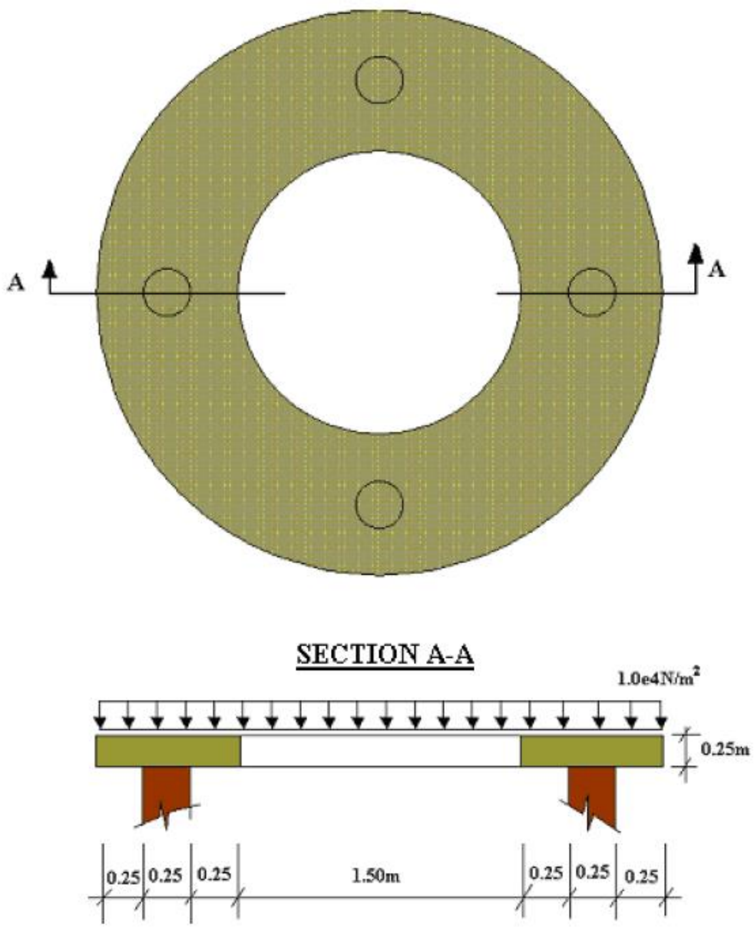


Figure 9   rotations, stresses and final shape of the plate (exaggerates x1000)

Exercise 3: Thick circular plate with internal hole

The figure shows a reinforced concrete plate supported on four columns, submitted to its dead weight and a uniform load. Analyze the structural behavior using the theory of thick plates of Reissner-Mindlin. Use triangular elements of Reissner-Mindlin with six nodes and reduced integration.



Data

$$\text{Concrete}\begin{cases} E = 3.0e10\,\dfrac{N}{m^2} \\ \nu = 0.2 \\ \gamma = 2.4e4\,\dfrac{N}{m^3} \end{cases}$$

Solution:

A reinforced concrete plate supported on four columns, submitted to its dead weight and a uniform load of 1E4 N/m2

Grid:

In GID, an unstructured mesh of triangles was constructed for the simulation.

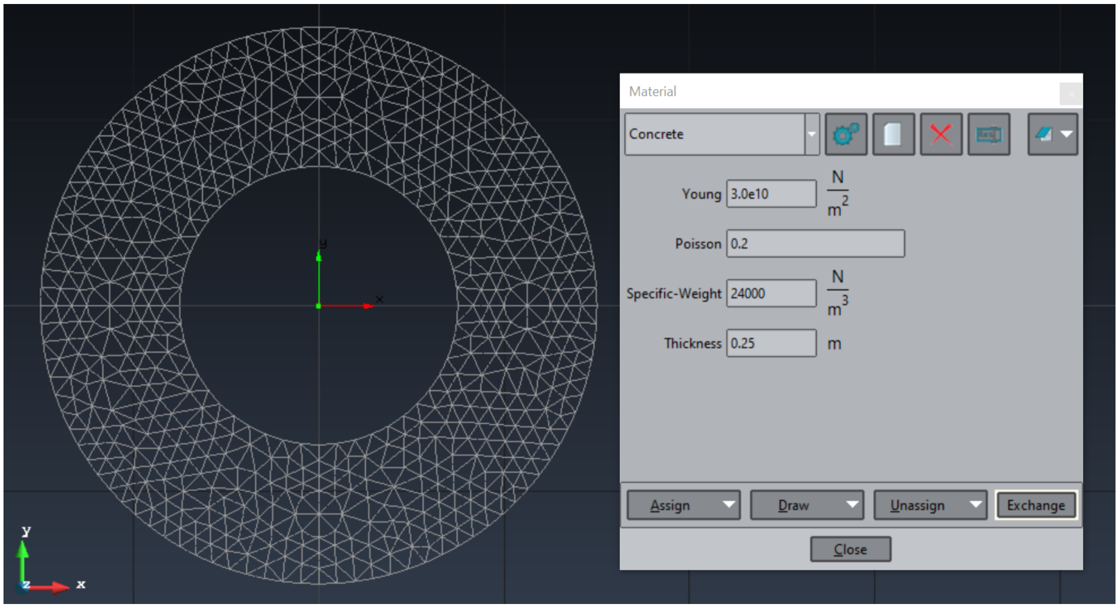


Figure 10 2D mesh constructed along with the material properties assigned. Num. of Triangle elements=1.146, Num. of nodes=2.436.

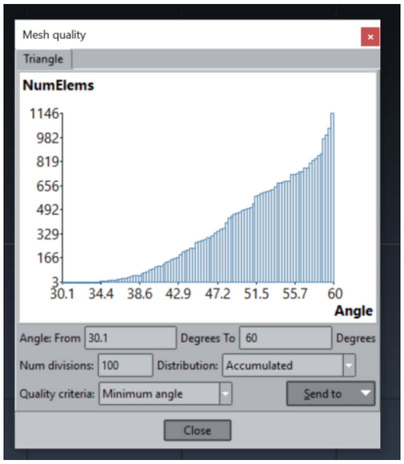
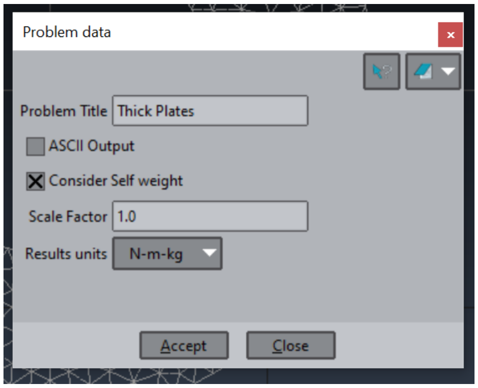
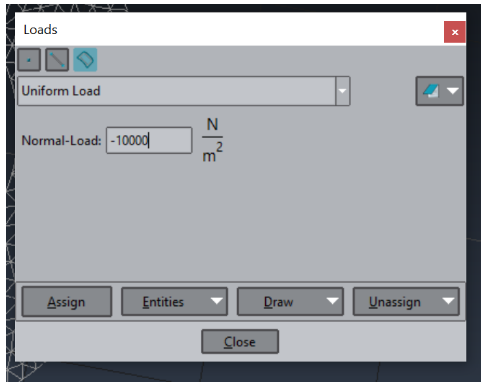


Figure 11 Mesh quality is intermediate as expected for an unstructured grid of triangles. However, no angles are below 30 degrees.

The proper self weight and uniform load were applied to the plate.





As well as the zero displacement boundary conditions

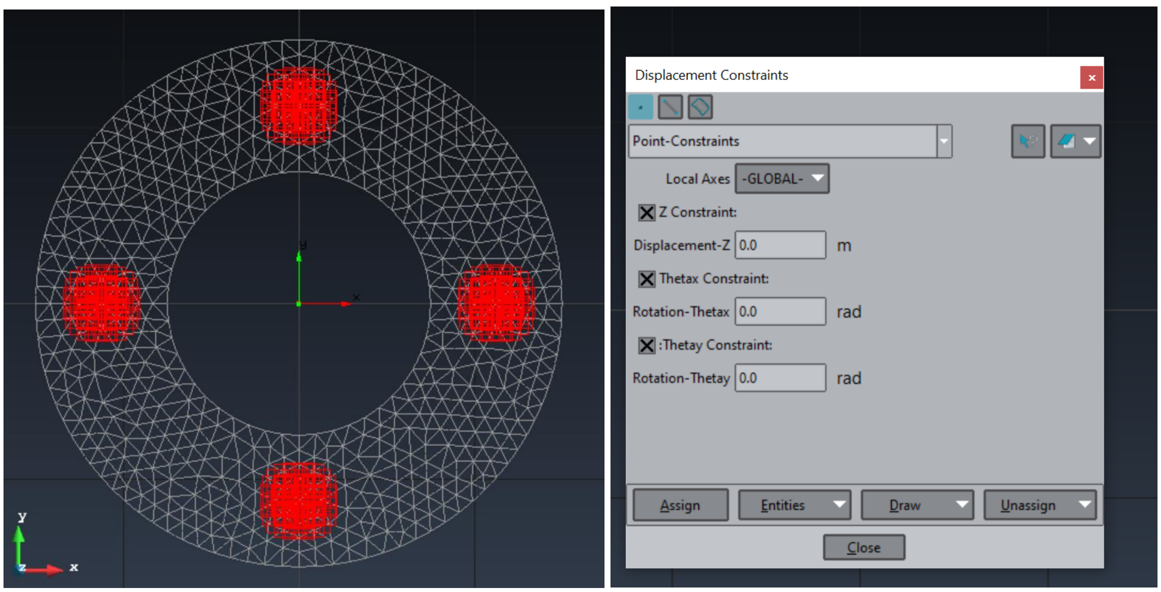


Figure 12 zero displacement constraints (in all directions) applied where the columns hold the structure

The simulation was carried out using triangular Reissner-Mindlin with six nodes and reduced integration.

Structural behaviour

The deformations experienced are maximum in the vertical direction at the mid distance between support and in the radially most external regions. Displacements of up to 2.3E-5 meters are predicted.

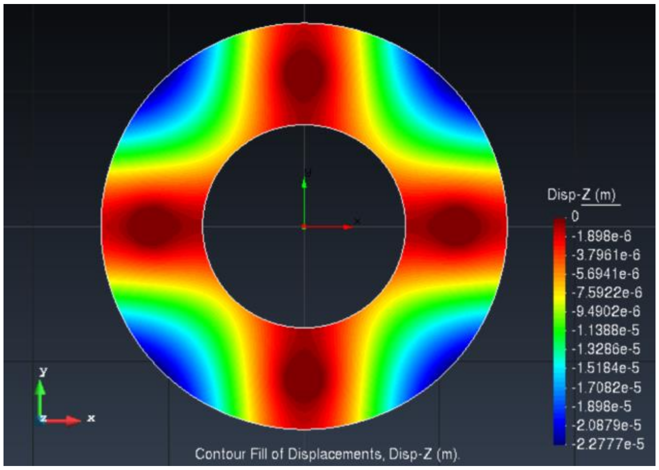


Figure 13   Vertical displacements (in the z direction) due to self-weight and the applied uniform load.

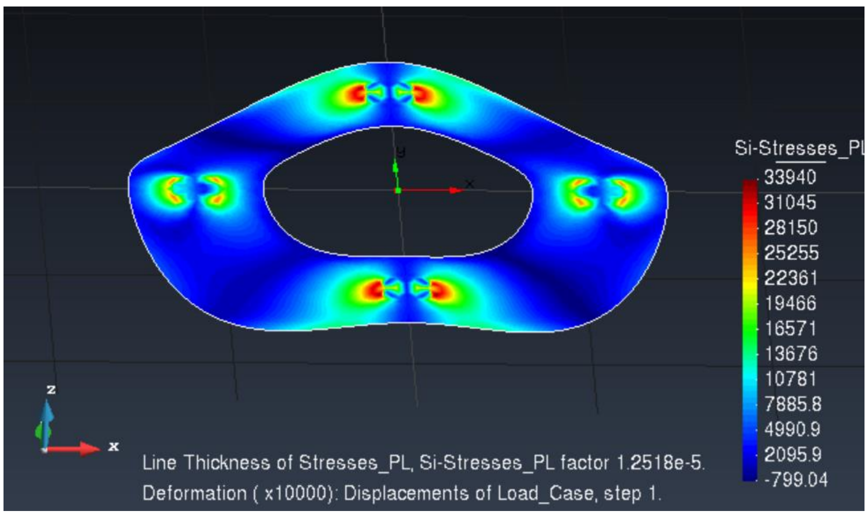


Figure 14   final shape of the plate under self-weight and uniform load (displacements exaggerated 10,000 times)

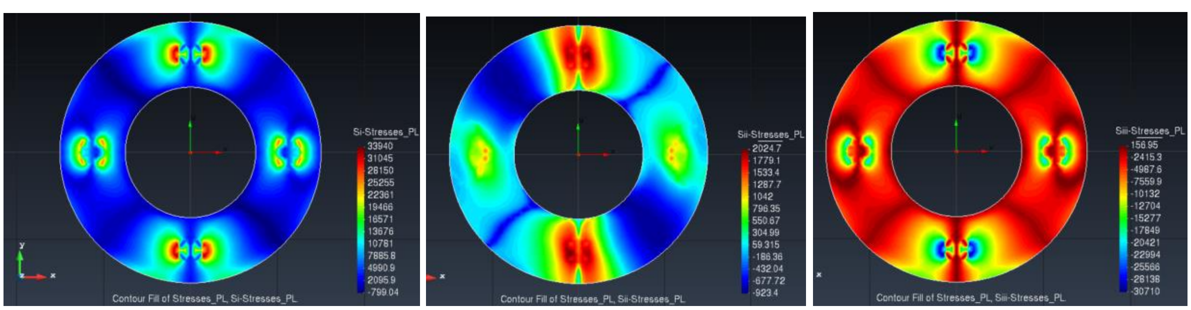


Figure 15   Stresses PL – from left to right Si, Sii and Siii