Computational Structural Mechanics and Dynamics

Practice 3

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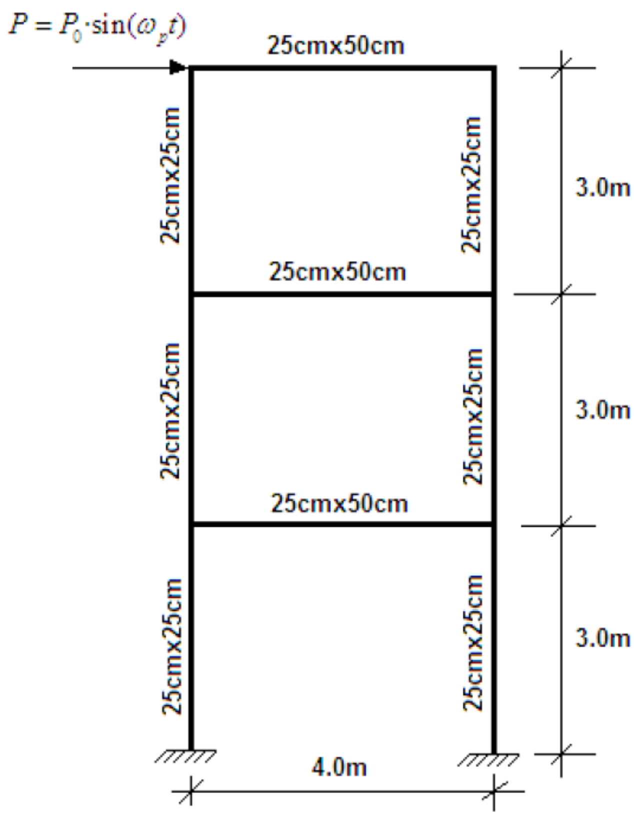
Exercise 1: Plane Frame

Calculate the natural frequencies and modes of the plane frame in the figure. Perform a modal analysis and direct integration. Use a dynamic load frequency with the values $\omega_p = 0.75\omega_1$, $1.0\omega_1$ y $1.25\omega_1$, where ω_1 is the principal natural frequency.



Data

$P_0 = 50$ kN

$$\text{Concrete} \begin{cases} E = 3.0e10 \ \dfrac{\text{N}}{\text{m}^2} \\ \nu = 0.2 \\ \gamma = 25 \dfrac{\text{kN}}{\text{m}^3} \end{cases}$$

Solution

Grid & material properties:

A very simple beam element model was constructed with 30 elements and 29 nodes. The properties of the elements are assigned as per the problem (concreted properties, with E = 3e10 N=m2, Poisson Ratio = 0.2, spec. weight= 25 kN/m3).

Boundary conditions

Nodes 8 and 1 (columns reashing the floor) were prescribed with zero displacement in X and Y. It is assumed they are fixed to the floor.

Node 29 is prescribed a force of 50kN in the x direction, modulated by a sin(wt) function. More details on w below, the frequency, below.

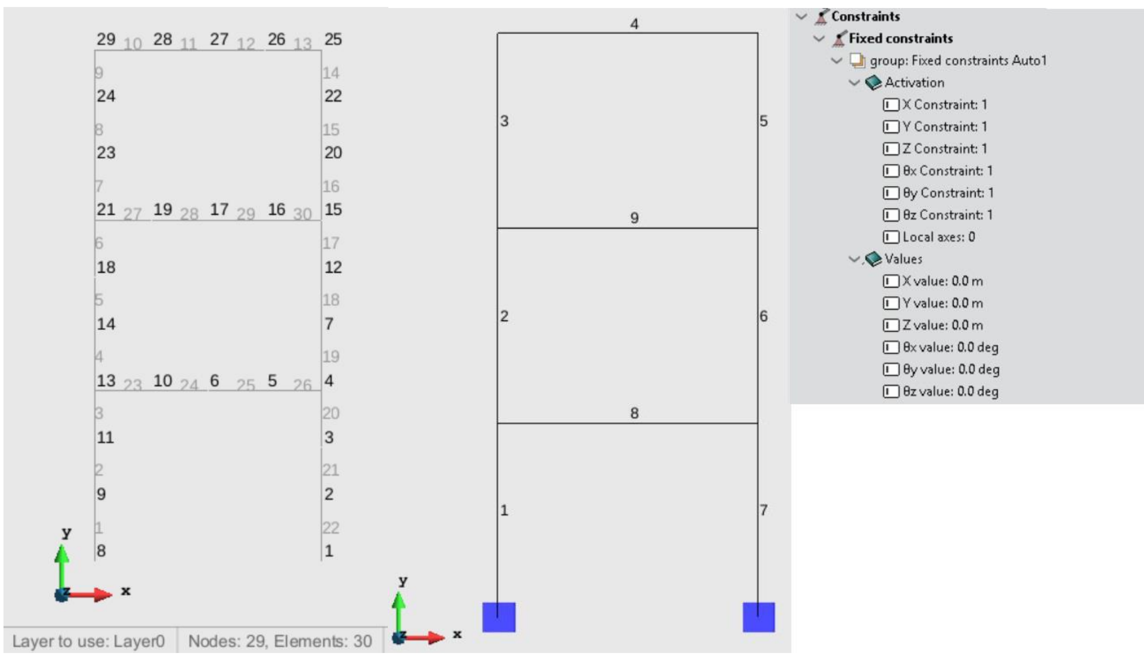


Figure 1 beam structure (left) and displacement constraints applied (right)

Determine the natural frequency w_p

In order to determine the natural frequencies of the structure, a Modal Analysis was carried out using General data ► Dynamic analysis data ► General ► Type: Modal analysis.

The table below shows the results of the modal analysis. There are two modes that have a large participation in the direction 49.75 Hz and 146.8 Hz. The former has the larger participation and it is therefore used going forward as the ‘natural’ frequency in the x-direction (w_p).

Mode	Freq [Hz]	Mass_x [%]	Mass_x [Kg]	Mass_y [%]	Mass_y [Kg]	Mass_z [%]	Mass_z [Kg]
1	28.38	0	-	0	-	3,565,000	5
2	48.78	0	-	0	-	0	-
3	49.75	11,040,000	17	0	-	0	-
4	56.73	0	-	58,050,000	87	0	-
5	77.29	0	-	0	-	3,003,000	4
6	108.7	0	-	0	-	178,700	0
7	141.3	0	-	0	-	0	-
8	146.8	8,188,000	12	0	-	0	-
9	164.2	0	-	5,149,000	8	0	-
10	224.9	0	-	0	-	0	-

Total	Mass	[Kg]=	66918876.48	
Modal	Participation	X	[%]=	28.74
Modal	Participation	Y	[%]=	94.44
Modal	Participation	Z	[%]=	10.08

Table 1 results of the modal analysis

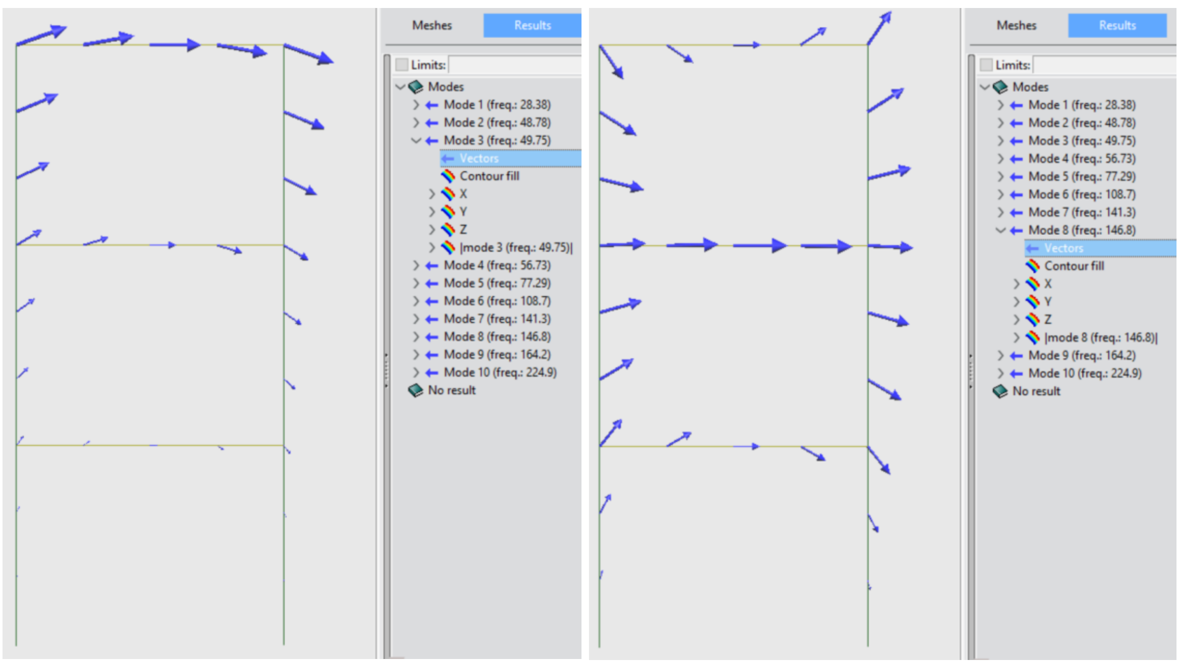


Figure 2 displacements associated to the two main natural frequencies in the x-direction (49.75Hz and 146.8Hz)

External forcing

Now the structure is simulated dynamically using three frequencies: 0.75wp, wp and 1.25 wp. The idea is to assess the structural response to different stimuli and show that an external force with that pushes with the same frequency as the natural frequency of the structure generates the largest displacements. This phenomon is known as resonance, and has led to catastrophic failures of structures, one of the most well known being the Tacoma Narrows bridge incident in 1940, where mild wind gushes blowing with frequency similar to that of the natural frequency of the bridge led to its total collapse of it.

Figure 3 Tacoma Narrows bridge failure incident

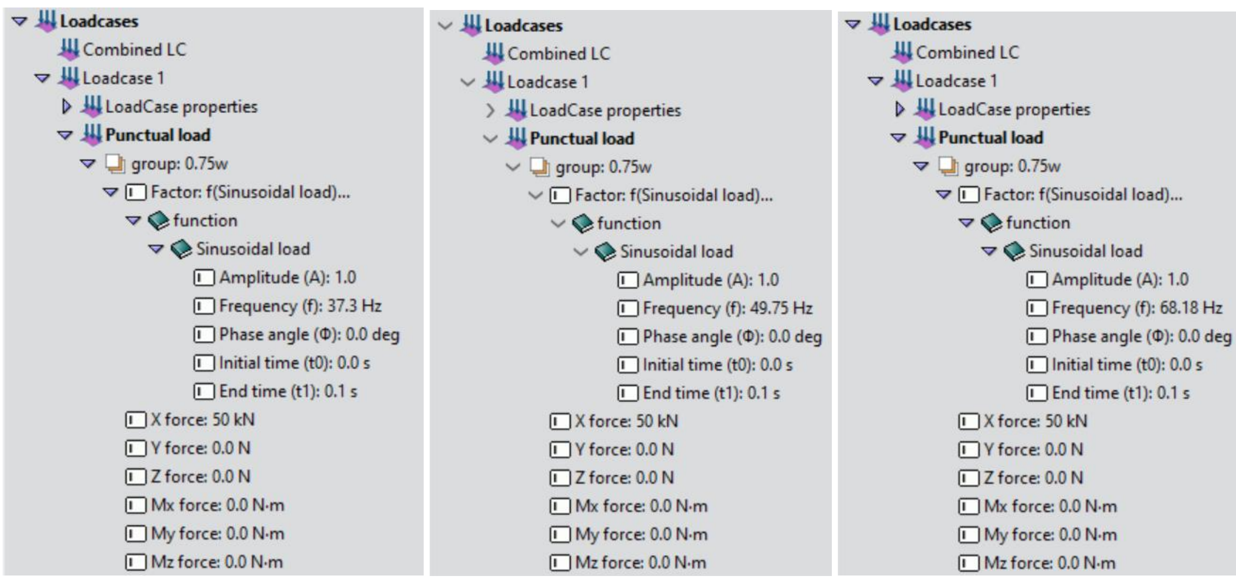


Figure 4 load cases for simulation with 0.75wp, wp and 1.25wp

Time step chosen should be approximately 20 times smaller than 1/wp. In this case, taking the largest frequency to determine the timestep for all the runs, we get 1/wp~0.014 seconds. So Δt=0.001 seconds was used to be on the safe side. This time step was used for all the simulations also to have data output with the same frequency too, making comparisons easier.

Results

As can be observed in the following figure. There are many facts worth noting.

- The amplitude of the displacement, when the external forces has the frequency equal to the strongest x direction natural frequency, tends to grow over time. In fact we ran another case with 1 second simulation and the amplitudes kept increasing still. This would continue to happen until either the amplitude of the system reaches the amplitude of the force or the system braeks completely (ceases to be elastic and deforms plastically).
- The frequency of the response is pure, not affected by other frequencies.

- The amplitudes when stimulated with 0.75wp or 1.25wp are smaller and remain bounded over time.
- The response when stimulated with 0.75wp or 1.25wp seem to show other frequencies is affected y other frequencies as observes by the dip in amplitudes at 0.08 seconds and the posterior regaining of amplitude, which is lost again at 0.16, and so on.

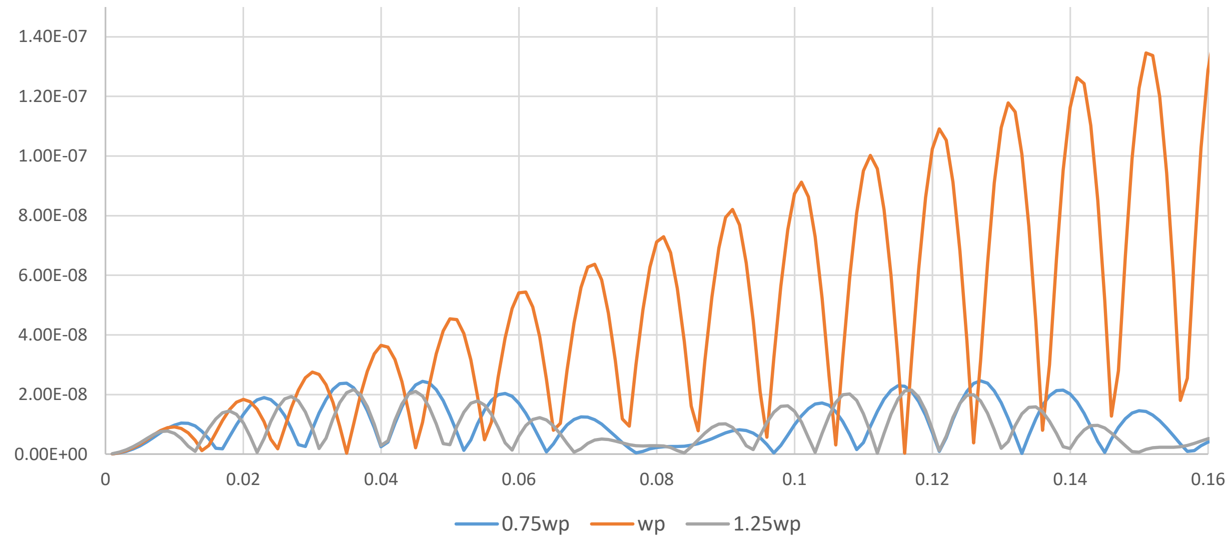


Figure 5 x-displacements in meters vs time in seconds of the upper left corner of the structure, for the three frequencies of stimulation.

A pictures of compressive and shear forces are shown below. Notice maximum shear occurs where the external force is applied.

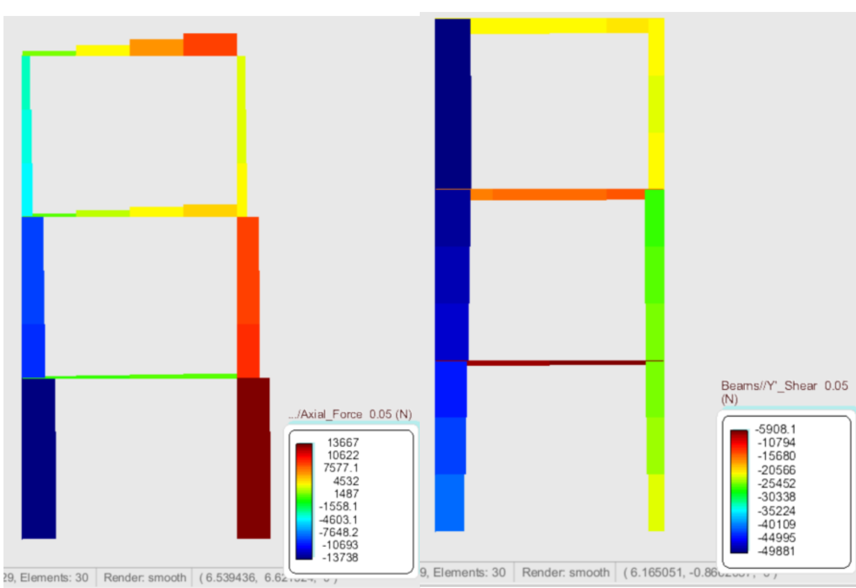
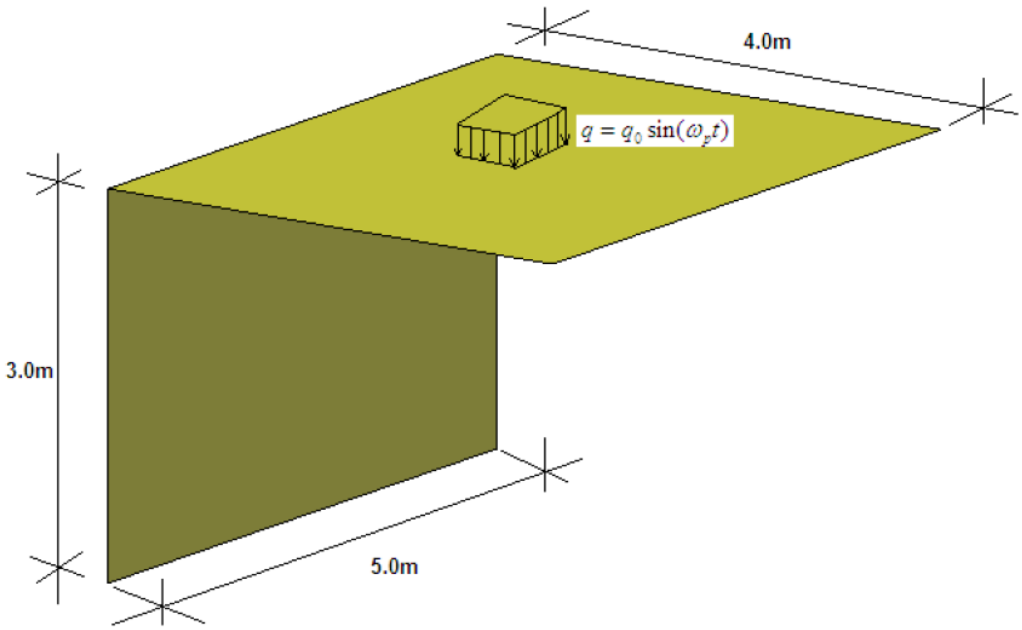


Figure 6 maximum stresses experienced.

Exercise 2: Spatial shell

Calculate the natural frequencies and modes of the spatial shell in the figure. Perform a modal analysis and direct integration. Use a dynamic load frequency with the values $\omega_p = 0.75\omega_1$, $1.0\omega_1$ *y* $1.25\omega_1$, where ω_1 is the principal natural frequency.



Data

$$q_0 = 50 \ \frac{\text{kN}}{\text{m}^2}$$

$$\text{Concrete} \begin{cases} \text{E} = 3.0\text{e}10 \ \frac{\text{N}}{\text{m}^2} \\ \nu = 0.2 \\ \gamma = 25 \frac{\text{kN}}{\text{m}^3} \\ \text{t} = \ 0.30 \text{ m} \end{cases}$$

Solution

Grid and problem conditions

Structured grid with 280 elements and 165 nodes was constructed to model the problem. The properties of the elements are assigned as per the problem (concreted properties, with E = 3e10 N=m2, Poisson Ratio = 0.2, spec. weight= 25 kN/m3). The structure was prescribed with zero displacements on edge 6, which were the structure is 'attached' to the ground somehow.

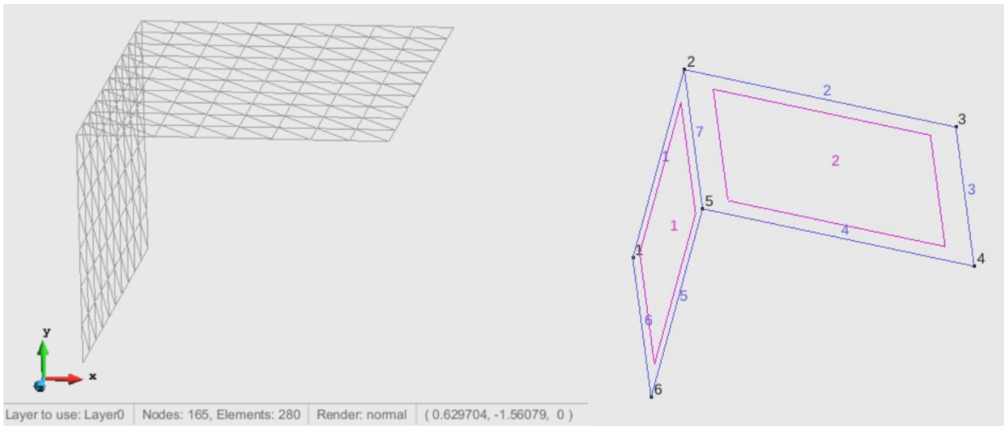


Figure 7 3D view of the grid

Determine natural frequency

Following the same procedure as in part 1, modal analysis was carried out with Ramseries professional. The next table shows the 10 most important frequencies. The most important frequency in the y direction is 4.525Hz, although other secondary y-direction frequencies are not negligible (12.65Hz and 52.16Hz). For simulating the external sinusoidal force, only the 4.52Hs frequency is used.

Mode	Freq [Hz]	Mass_x [%]	Mass_x [Kg]	Mass_y [%]	Mass_y [Kg]	Mass_z [%]	Mass_z [Kg]
1	4.525	6,035	23	7,942	30	0	-
2	8.874	1	0	0	0	7,750	29
3	12.65	15,800	59	2,480	9	0	0
4	20.3	0	0	0	0	225	1
5	52.16	512	2	2,273	8	0	0
6	57.88	111	0	531	2	0	0
7	74.42	1	0	0	0	390	1
8	85.21	1	0	0	0	12,660	47
9	106.8	1,875	7	91	0	3	0
10	119	0	0	0	-	801	3

Total	Mass	[Kg]=	26767.55059	
Modal	Participation	X	[%]=	90.93
Modal	Participation	Y	[%]=	49.75
Modal	Participation	Z	[%]=	81.55

Table 2 results of the modal analysis

The first three vibration modes of the modal analysis are shown below

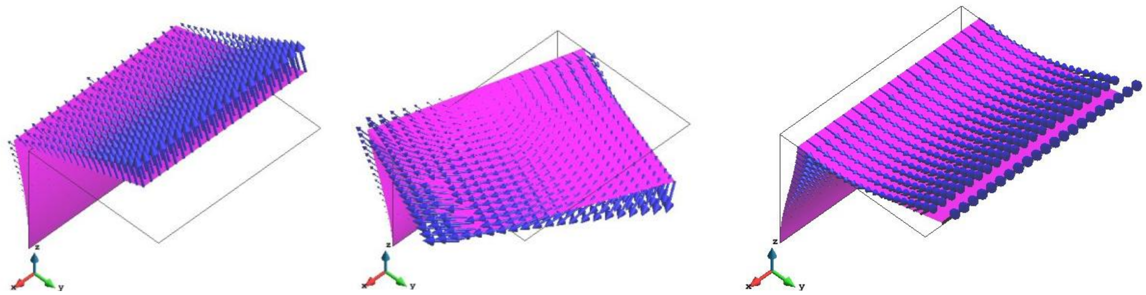


Figure 8   First three vibration modes from the modal analysis

Simulation of resonance

The sinusoidal load of 50kN/m2 was applied, modulated by the sin(wp*t) function, on the top of the structure. The time step chosen should be at least 20 times smaller than 1/wp. In this case, taking the largest frequency to determine the timestep for all the runs, we get 1/wp~0.008 seconds. So Δt=0.01 seconds was considered good enough.

The maximum displacement vectors can be observed in the following figure:

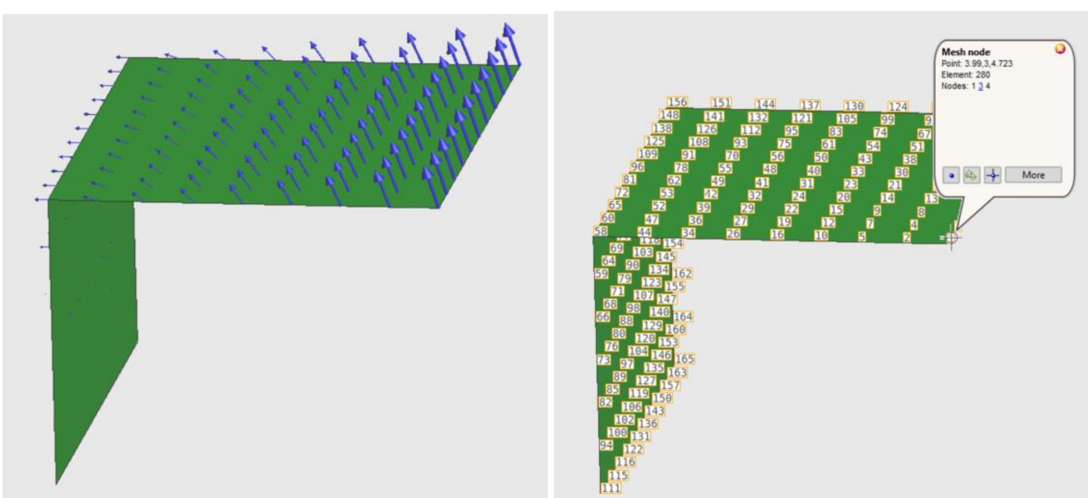


Figure 9   maximum displacements observed during a 0.5 second simulation run (left). Node at which displacements are plotted (right)

Taking the front right corner of the structure as indicated to obtain the time series of the displacements we observe the following displacement amplitudes:

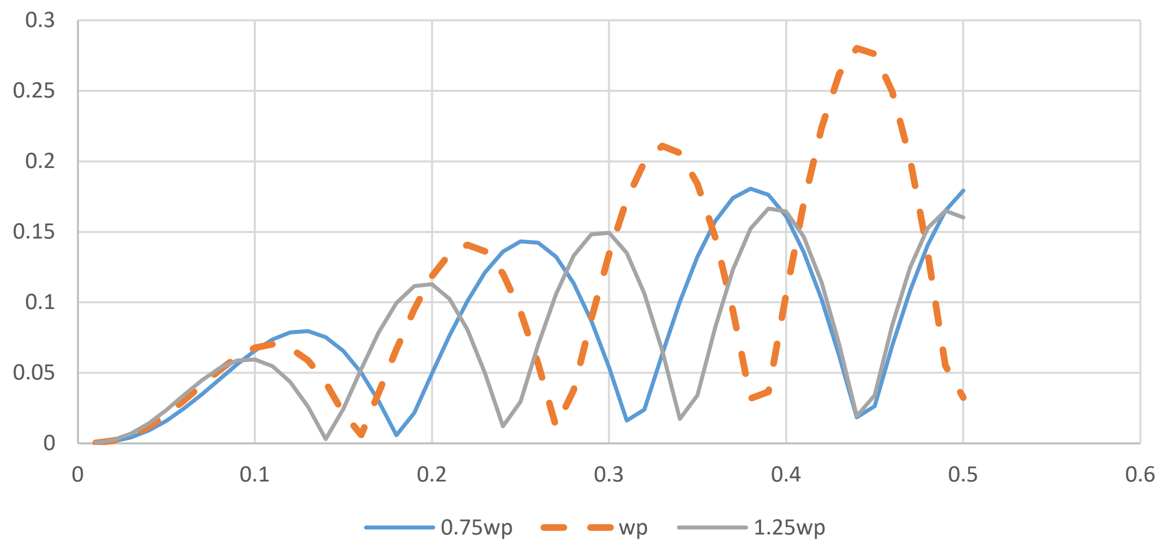


Figure 10 Amplitudes simulated on the front left corner.

The conclusion is similar to the previous section. When stimulated with a sinusoidal load of frequency equal to the natural frequency the system enter resonance and the amplitudes grow over time without bound until something happens, either the system breaks or the amplitude of the structure reaches that of the external load.