FINITE ELEMENTS IN FLUIDS

Assignment 3: 2D Steady Convection and Diffusion

1. Implementation of Galerkin Least Squares method and Triangular Quadratic Elements

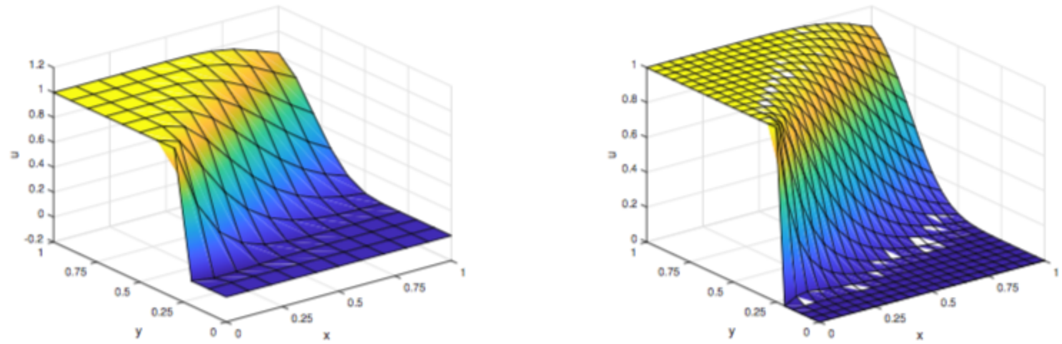
For this part we are going to implement GLS in “FEM_system.m” such as:

```
elseif method == 3
% GLS
aux = N_ig*Xe; % Variable aux formada per nodes*shape_func
RT_ig = RT(aux); % Definim el Reaction Term RT
Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny)...
    N_ig'*RT_ig*N_ig + tau*((((ax*Nx+ay*Ny)-nu*(Nxx+Nyy)+...
    RT_ig*N_ig)'*((ax*Nx+ay*Ny)-nu*(Nxx+Nyy)+RT_ig*N_ig))))...
    *dvolu;
f_ig = ST(aux); %Afegim el Source Term i el force vector
fe = fe + (N_ig+tau*((ax*Nx+ay*Ny)-nu*(Nxx+Nyy)))'*(f_ig*dvolu);
```

And then we introduce the Triangular Quadratic elements in the subroutine called “ShapeFunc.m” such as:

```
    %IMPLEMENTATION OF THE TRIANGULAR QUADRATIC ELEMENT
    elseif p == 2
        N = [ xi.*(2*xi-1), eta.*(2*eta-1), (1-xi-eta).*(2*(1-xi-eta)-1),...
            4*xi.*eta, 4*eta.*(1-xi-eta), 4*(1-xi-eta).*xi];

        Nxi = [ 4*xi - 1, zeros(size(xi)), 4*eta + 4*xi - 3,...
            4*eta, -4*eta, 4 - 8*xi - 4*eta];

        Neta = [ zeros(size(xi)), 4*eta - 1, 4*eta + 4*xi - 3,...
            4*xi, 4 - 4*xi - 8*eta, -4*xi];

        N2xi = [ 4*ones(size(xi)), 0*ones(size(xi)), 4*ones(size(xi)),...
            0*ones(size(xi)), 0*ones(size(xi)), -8*ones(size(xi))];

        N2eta = [  0*ones(size(xi)), 4*ones(size(xi)), 4*ones(size(xi)),...
            0*ones(size(xi)), -8*ones(size(xi)), 0*ones(size(xi))];



    else
```

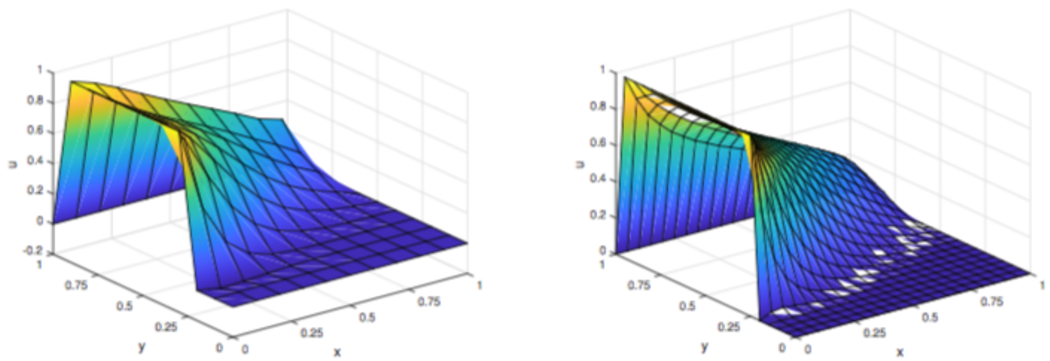
We can now comment the results:



In this previous figure, we can see on our left, GLS with 10 linear elements, and in our right, GLS with same 10 elements but in this case quadratic. It's obvious that the accuracy is much larger in the second case starting from the left. But no big difference is appreciated apart from the one said.

Implementation of Dirichlet Boundary Condition equal to zero

In this part we will modify the code so we can get zero on the value on the outlet Dirichlet boundary conditions.



As we can see above, we got GLS with 10 elements on both cases (linear elements on left and quadratic on the right). The results show that, as it happened in the previous figure, both seem to have a correct approximation of the solution but the quadratic one shows more accuracy.