



**Finite element in fluid**

**Cavity Assignment**

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Using mixed finite elements in solution of stationary Stokes flow :

$$-\nu \nabla^2 v + \nabla p = b \quad \in \Omega$$

$$\nabla \cdot v = 0 \quad \in \Omega$$

$$v = 0 \quad \Gamma$$

Q2Q1 → Continuous biquadratic velocity and continuous bilinear pressure.

Q2Q0 → Continuous biquadratic velocity and discontinuous pressure.

Q1Q1 → Continuous bilinear velocity and continuous bilinear pressure.

Q1Q1\* → Continuous bilinear velocity + cubic bubble function and continuous bilinear pressure.

Q2Q1 element, which satisfy LBB condition, deliver good result for velocity and pressure.

Q1Q1 and Q2Q0 elements that do not satisfy LBB condition shows a spurious pressure response, but the result for velocity is acceptable.

When LBB condition does not satisfy, we should use stabilization techniques.

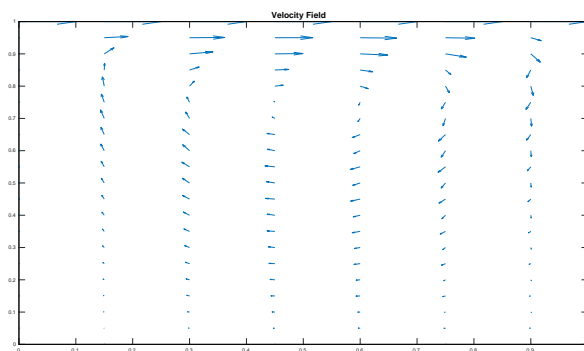
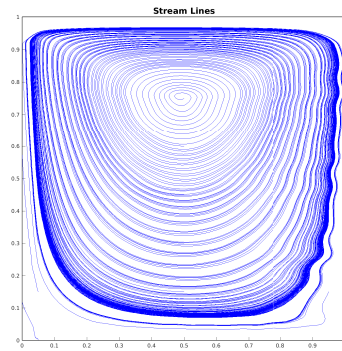
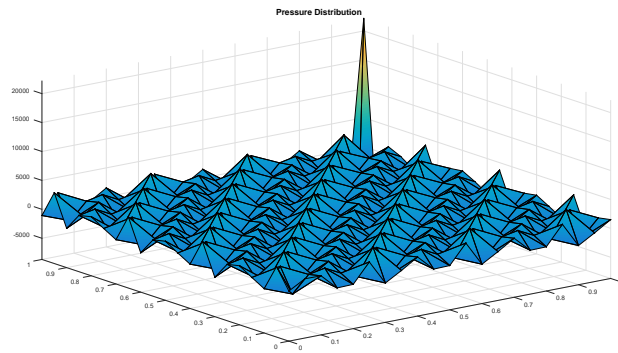


Figure 1: Q1Q1

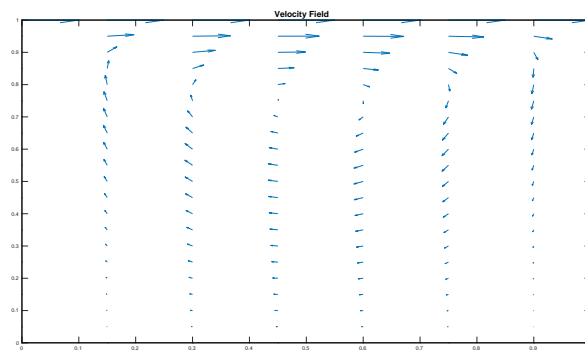
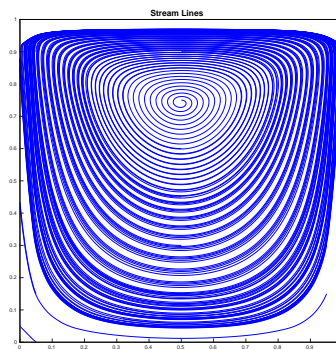
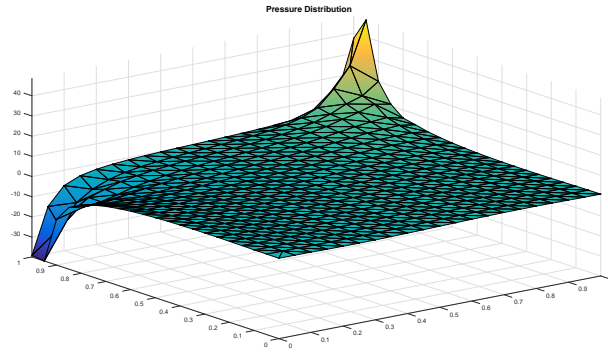


Figure 2: Q1Q1\*



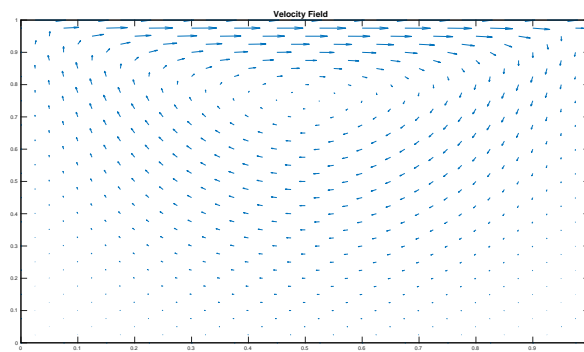
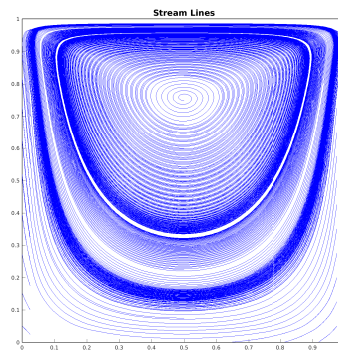
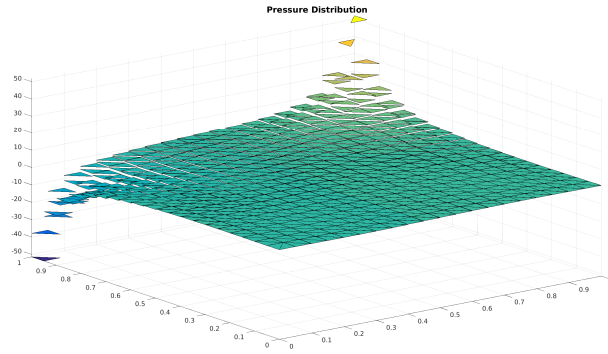


Figure 3: Q2Q0

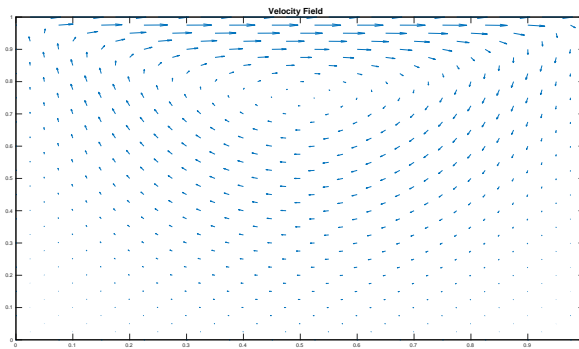
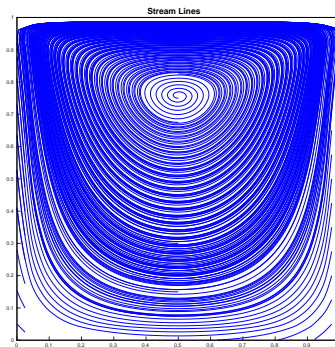
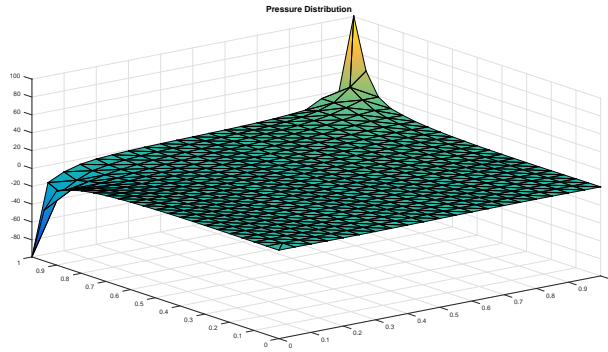


Figure 4: Q2Q1

When we want to obtain high value for velocity near the wall, it is good to use adapted mesh that we have smaller elements.

The result for adapted mesh is nice for velocity and pressure.

Adapted mesh exhibit high variation near the wall.

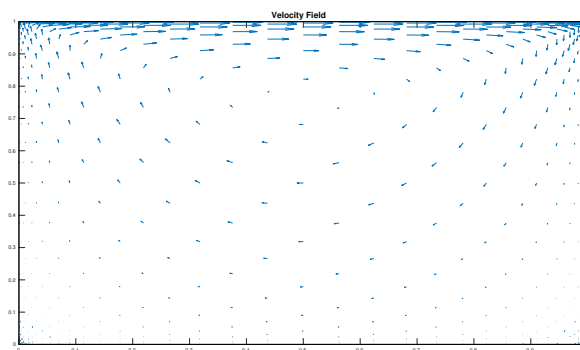
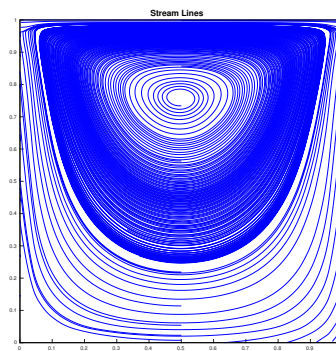
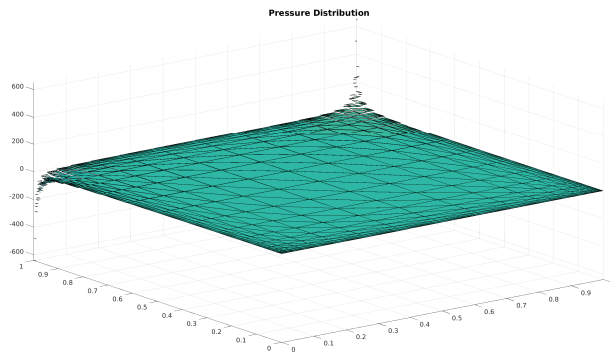


Figure 5: Q2Q0

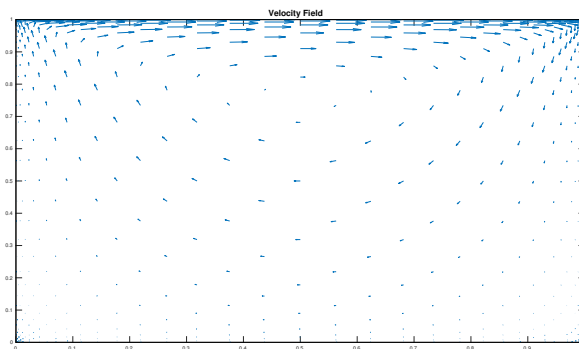
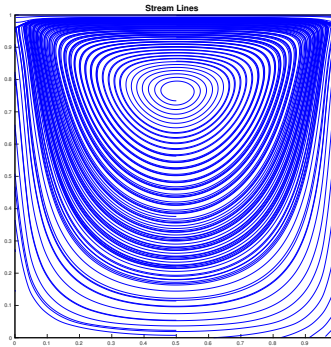
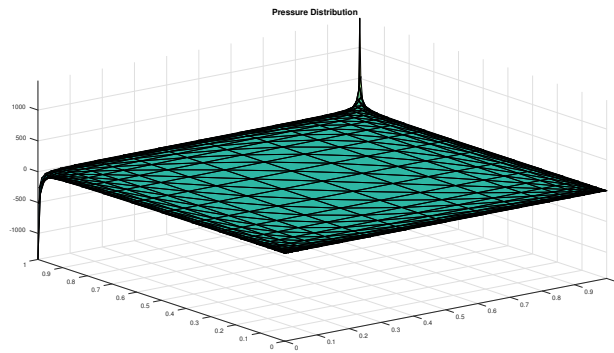


Figure 6: Q2Q1

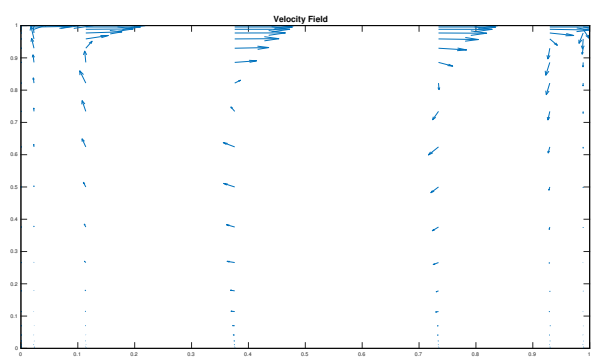
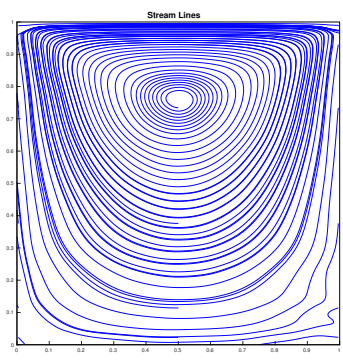
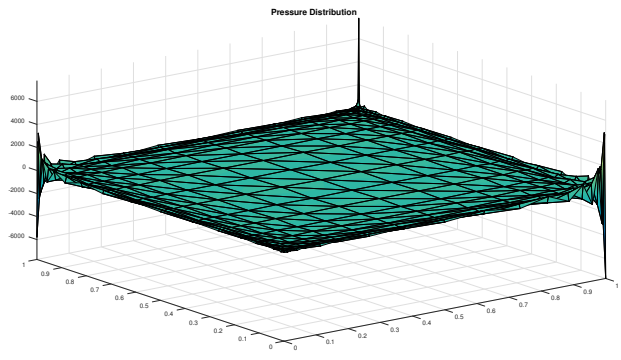


Figure 7: Q1Q1

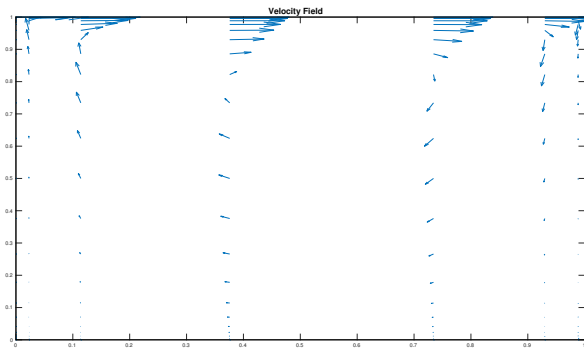
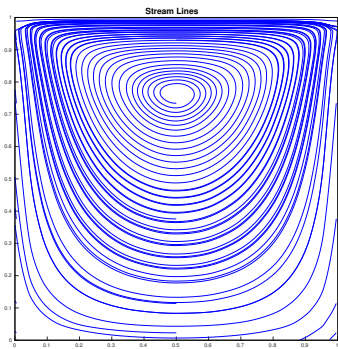
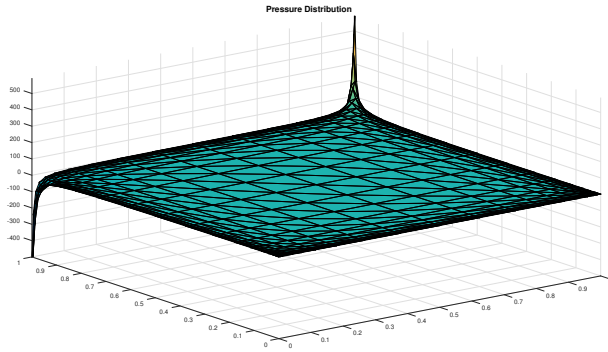


Figure 8: Q1Q1

$$-\nu\Delta\mathbf{v} + \nabla\mathbf{p} = \mathbf{f}$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\mathbf{L}_1(\bar{\mathbf{v}}, \mathbf{p}) := -\nu\Delta\mathbf{G} + \nabla\mathbf{p}$$

$$\mathbf{L}_2(\bar{\mathbf{v}}, \mathbf{p}) := \nabla \cdot \bar{\mathbf{v}}$$

Galerkin

$$\int \bar{\mathbf{w}} \cdot \mathbf{L}_2(\nabla \cdot \mathbf{p}) = \bar{\mathbf{f}}$$

$$\int \mathbf{q} \mathbf{L}_2(\bar{\mathbf{v}} \cdot \mathbf{p}) = 0$$

$$\int \nabla \bar{\mathbf{w}} : \nu \nabla \bar{\mathbf{v}} \, d\Omega - \int \rho \nabla \cdot \mathbf{w} \, d\Omega = \int \mathbf{w} \cdot \bar{\mathbf{f}} \, d\Omega$$

Stabilization

$$+ \int \tau_1 \mathbf{L}_1(\bar{\mathbf{w}}, \mathbf{q}) [\mathbf{L}_1(\bar{\mathbf{v}}, \mathbf{p}) - \bar{\mathbf{f}}] \, d\Omega$$

$$+ \int \tau_2 \mathbf{L}_2(\bar{\mathbf{w}}, \mathbf{q}) [\mathbf{L}_2(\bar{\mathbf{v}}, \mathbf{p}) - \bar{\mathbf{f}}] \, d\Omega$$

$$\begin{bmatrix} K + \bar{K} & -G + \bar{G} \\ G + \bar{G} & -L \end{bmatrix} \begin{bmatrix} \bar{V} \\ \bar{P} \end{bmatrix} = \begin{bmatrix} \bar{f} + F_1 \\ F_2 \end{bmatrix}$$

$$\bar{K} \leftarrow \Sigma \int [\mathbf{L}_1(\nu\Delta\bar{\mathbf{w}})(\nu\Delta\bar{\mathbf{v}})] \, d\Omega$$

$$\bar{G} \leftarrow \Sigma \int [\tau_1 \nabla \mathbf{q}(-\nu\Delta\bar{\mathbf{v}})] \, d\Omega$$

$$L \leftarrow \Sigma \int \tau_1 \nabla \mathbf{q} \cdot \nabla \mathbf{p} \, d\Omega$$

$$F_1 = \Sigma \int \tau_1 (-\nu\Delta\bar{\mathbf{w}}) \bar{\mathbf{f}} \, d\Omega$$

$$F_2 = \Sigma \int \tau_1 \nabla \mathbf{p} \cdot \bar{\mathbf{f}} \, d\Omega$$

$$\begin{bmatrix} K & -G \\ -G & -L \end{bmatrix} \begin{bmatrix} \bar{V} \\ \bar{P} \end{bmatrix} = \begin{bmatrix} \bar{f} \\ -F_2 \end{bmatrix}$$

$$\tau_1 = \alpha_0 \frac{h_e^2}{4\nu}$$

$$\alpha_0 = \frac{1}{3} \rightarrow \tau_1 = \frac{1}{3} \frac{h_e^2}{4\nu}$$

For GLS stabilization we can use the same degree for velocity and pressure and we have stable solution for element which does not satisfy LBB condition and we can use Galerkin formulation.

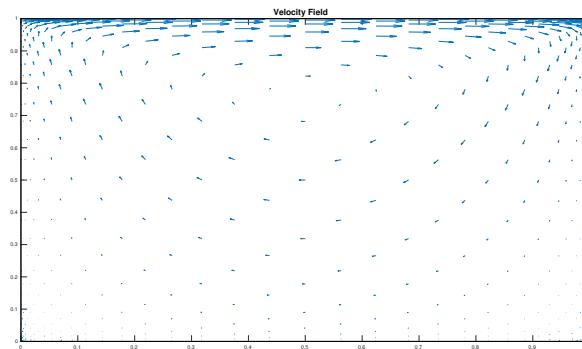
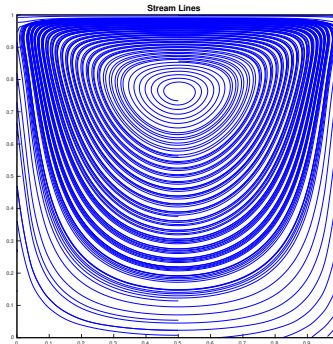
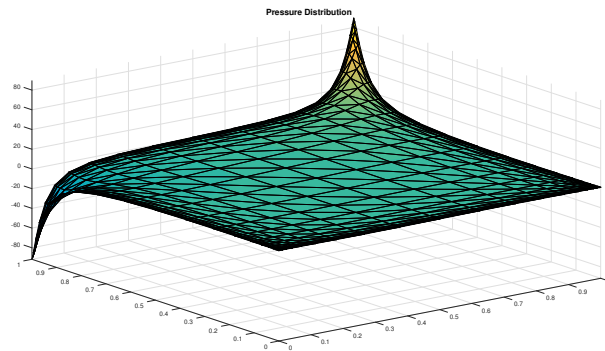


Figure 9: Q2Q1(GLS)



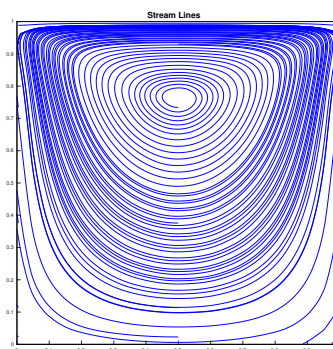
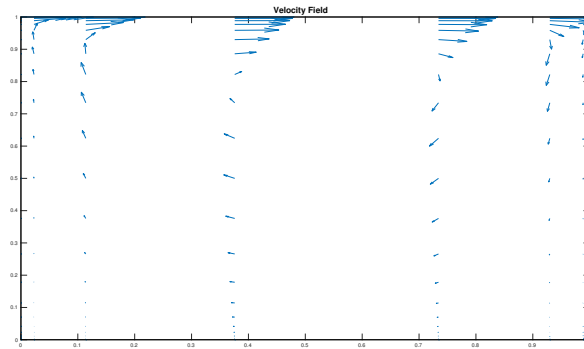
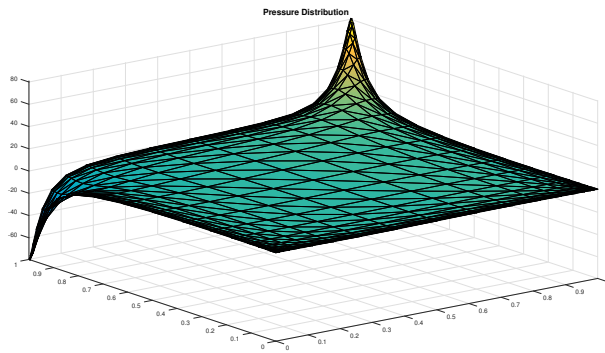


Figure 10: Q1Q1(GLS)

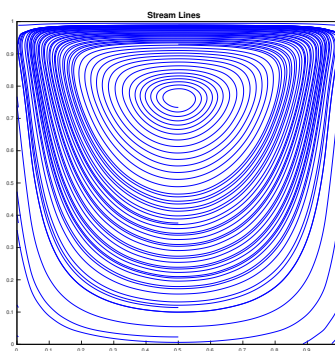
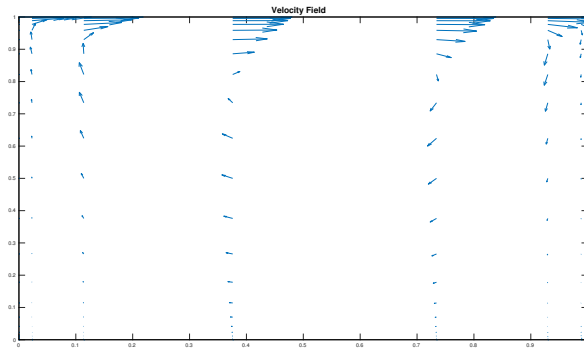
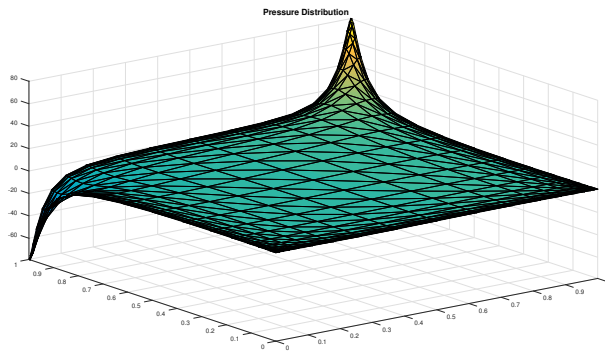


Figure 11: Q1Q1\*(GLS)

## Navier Stokes

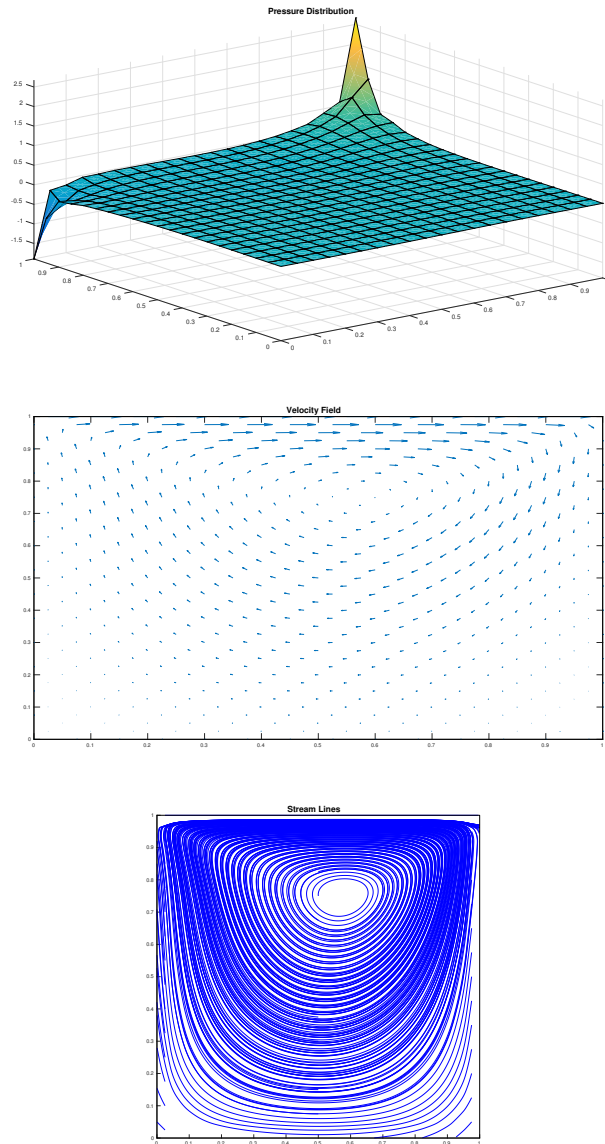


Figure 12:  $Re=50$

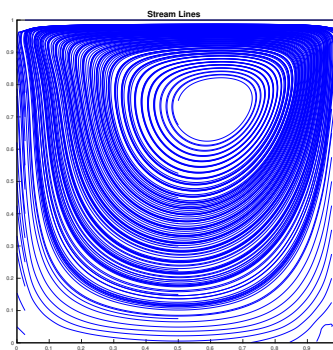
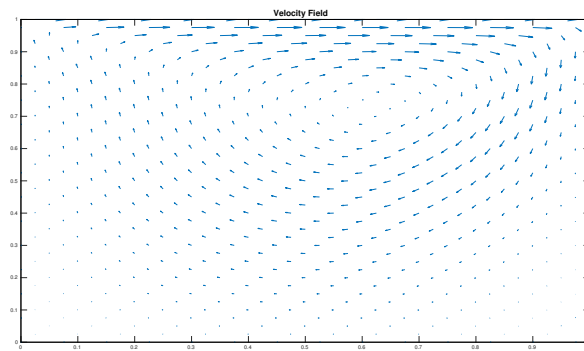
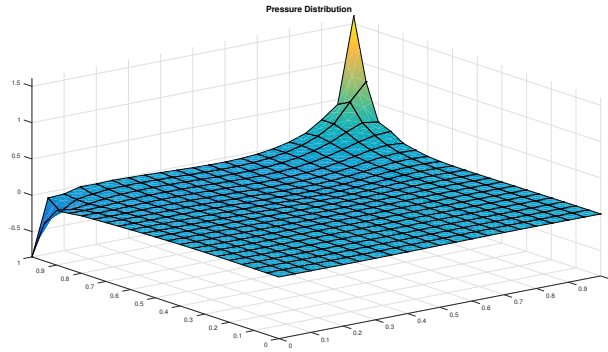


Figure 13:  $Re=100$

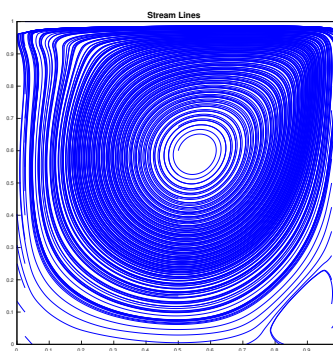
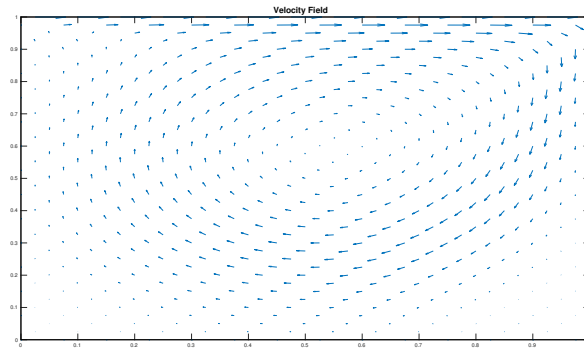
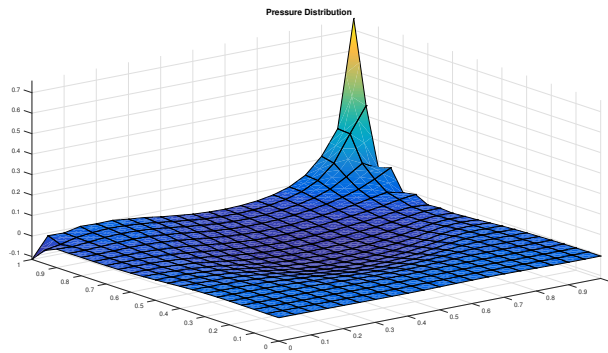


Figure 14:  $Re=500$

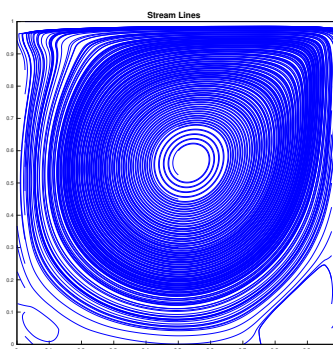
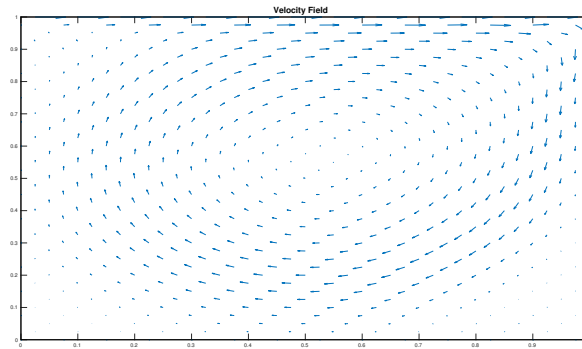
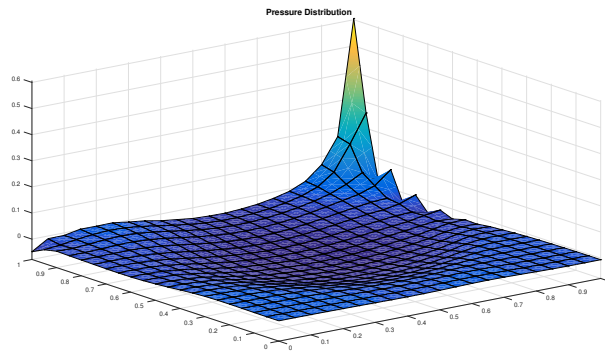


Figure 15:  $Re=1000$

For Navier stokes with  $Re = 50$ ,  $Re = 100$ ,  $Re = 500$  and  $Re = 1000$ . As Reynolds number increases boundary layer are more clear and velocity become sharper.

For Reynolds number in 100 and 100, we can use standard Galerkin finite element and stabilized formula should use for high value of Reynolds number.

For nonlinear equations, we can use Picard iterative techniques.

Number of iteration for  $Re = 50 \rightarrow 11$

Number of iteration for  $Re = 100 \rightarrow 14$

Number of iteration for  $Re = 500 \rightarrow 27$

Number of iteration for  $Re = 1000 \rightarrow 68$

We can find out when we increase number of iteration, Reynolds number will increase.