Steady Convection-Diffusion Problem

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In Steady Convection-Diffusion Problem part, we have learned several finite element methods like Galerkin Method, SU, SUPG and GLS. The following part will present the applications and the comparisons between these methods.

1 Galerkin Method

Compared with exact solution, the Galerkin method lacks diffusion. This method's truncation causes that it can only be used in specific situation. When Pelect number Pe<1, the Galerkin method works well. But if the Pelect number increases more than 1, meaning that the convection effect dominates over diffusion effect, oscillations will happen and the results will become unreasonable.

1.1 Pe<1

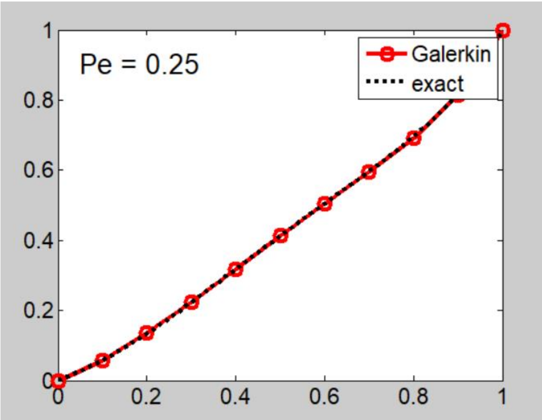

Figure 1. Pe=0.25(a=1, v=0.2, 10 linear elements)

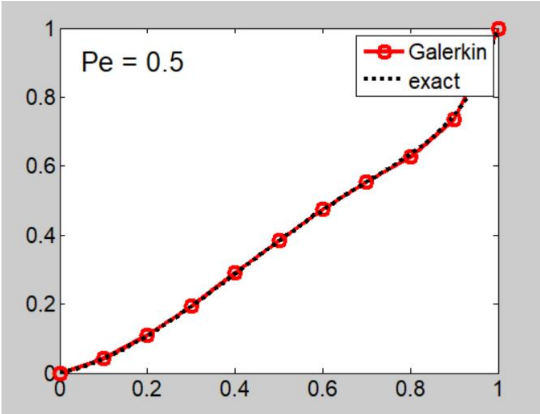

Figure 2. Pe=0.5(a=1, v=0.1, 10 linear elements)

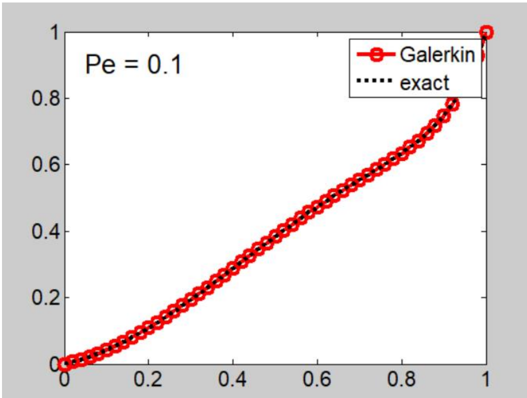

Figure 3. Pe=0.1(a=1, v=0.01, 50 linear elements)

From above cases, we can see that the Galerkin Method works very well when Pe<1. Now turning to the situation that Pe>1 as following.

1.2 Pe>1

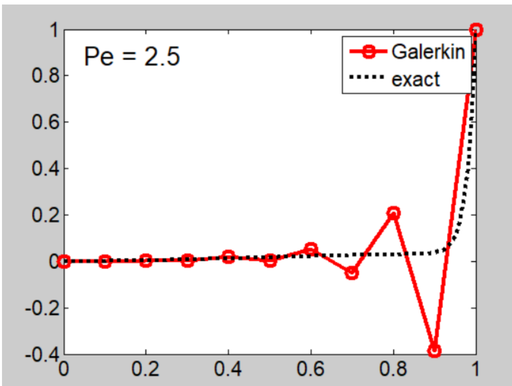

Figure 4. Pe=2.5(a=20, v=0.4, 10 linear elements)

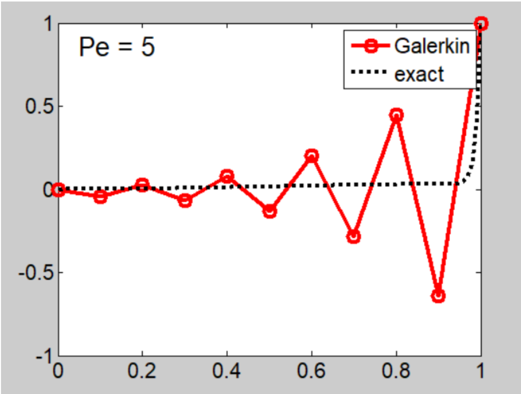

Figure 5. Pe=5(a=20, v=0.2, 10 linear elements)

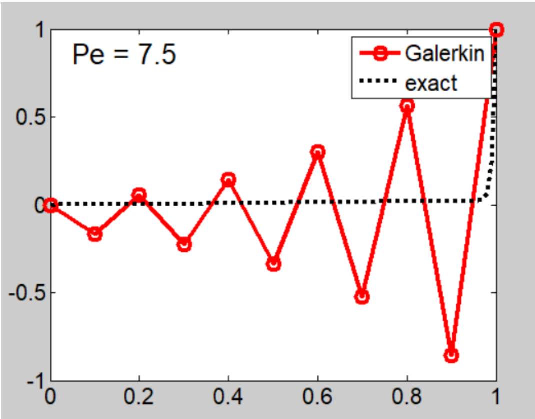

Figure 6. Pe=7.5(a=30, v=0.2, 10 linear elements)

From above three cases, we find that there are oscillations happening by using Galerkin Method when Pe>1. So other methods should be used in this situation.

2 SU Method

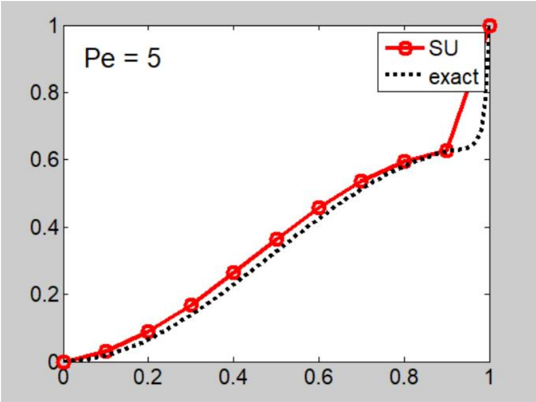

Figure 7. Pe=5(a=20, v=0.2, 10 linear elements)

From this result, we can find that the oscillation doesn't happen when Pe=5>1, but the result is not exactly equal to the exact solution. This is because that SU method doesn't work well when f is not a constant.

3 SUPG Method

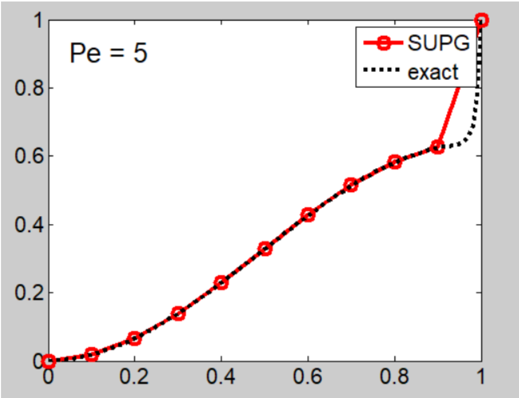

Figure 8. Pe=5 (a=20, v=0.2, 10 linear elements)

We can find that SUPG method also works very well because SUPG introduce less crosswind diffusion.

4 GLS Method

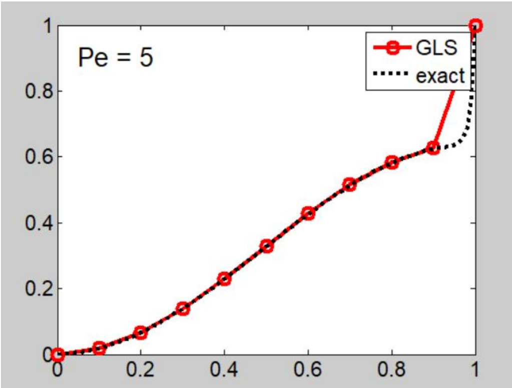

Figure 9. Pe=5 (a=20, v=0.2, 10 linear elements)

This result shows that the GLS method works very well too.

5 Effect of Stabilization Parameter (SUPG)

5.1 $\tau = 1$

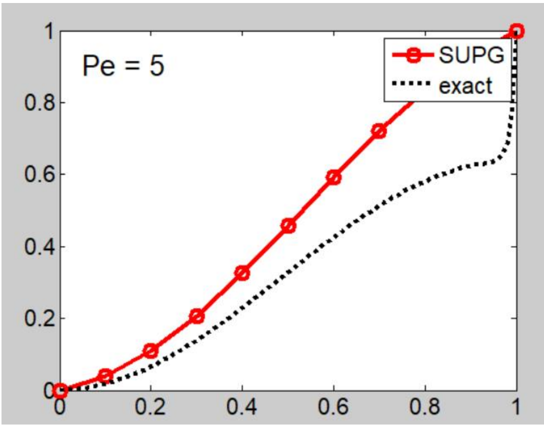

Figure 9. Pe=5 (a=20, v=0.2, 10 linear elements, $\tau = 1$)

From this case, we can see that the result computed by SUPG deviates the exact solution considerably. The cause of this phenomenon is that we have introduced excessive artificial diffusion when τ is bigger than the optimal magnitude, which makes the result wrong.

5.2 $\tau = 0.01$

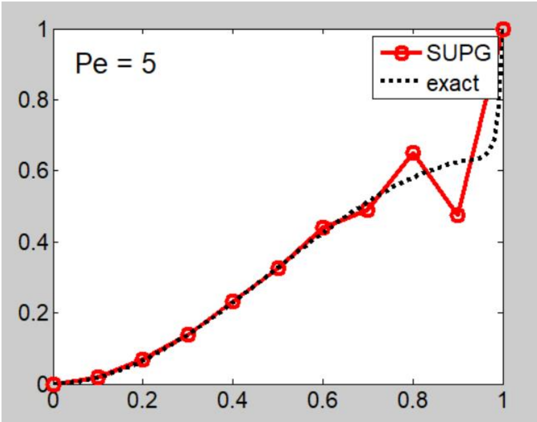

Figure 9. Pe=5 (a=20, v=0.2, 10 linear elements, $\tau = 0.01$)

From this picture, we can see that when x>0.6, oscillation happens. This is because that when τ is much smaller than the optimal magnitude, the effect of the added artificial diffusion is too weak to stop oscillation.

6 The changed code of SUPG and GLS from Galerkin

6.2 SUPG

```
Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig)+tau*a*a*(Nx_ig'*(Nx_ig)));
x = N_ig*Xe; % x-coordinate of the gauss point
s = SourceTerm(x,example);
fe = fe + w_ig*(N_ig'+tau*a*Nx_ig')*s;
```

6.3 GLS

```
Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig)+tau*a*a*(Nx_ig'*(Nx_ig))-a*tau*nu*Nx_ig'*(Nx2_ig));
x = N_ig*Xe; % x-coordinate of the gauss point
s = SourceTerm(x,example);
fe = fe + w_ig*(N_ig'+tau*a*Nx_ig'-tau*nu*Nx2_ig')*s;
```