Unsteady convection and convection-diffusion problem

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Part I

1．TG2 method

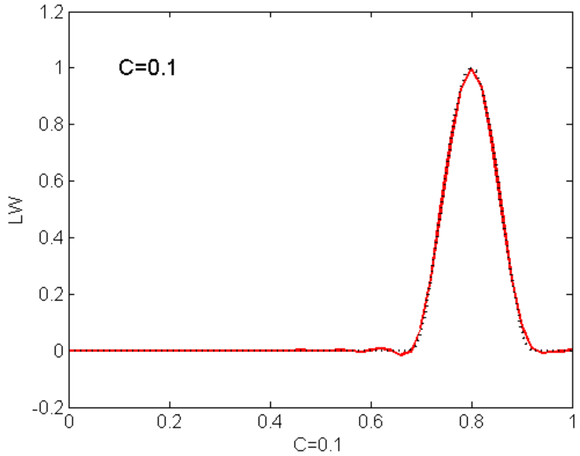

Figure 1. The result of TG2 with C=0.1

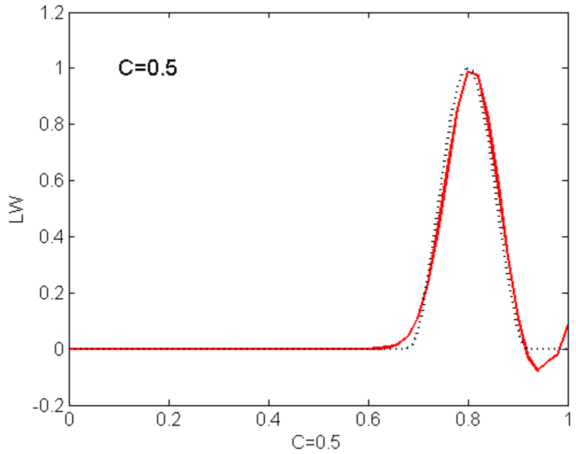

Figure 2. The result of TG2 with C=0.5

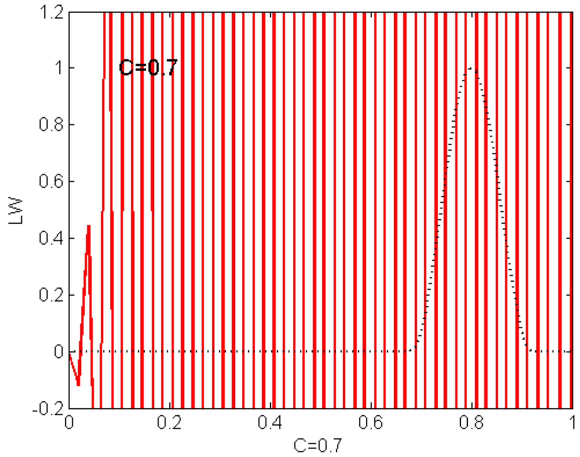

Figure 3. The result of TG2 with C=0.7

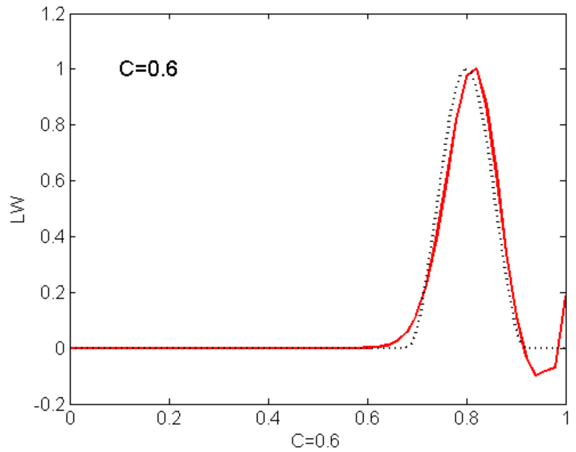

Figure 4. The result of TG2 with C=0.6

The TG2 method is conditionally stable. Only when $C^2 < \frac{1}{3}$, TG2 method will be reliable. And from above figures, when $C = 0.7$ whose square is larger than $\frac{1}{3}$, the result shows oscillations. When $C = 0.1$ and $C = 0.5$, the results are stable. But when $C = 0.5$, the result is not accurate. It has phase error. When decreasing the value of C to 0.1, the phase error decreases accordingly.

2. TG3 method

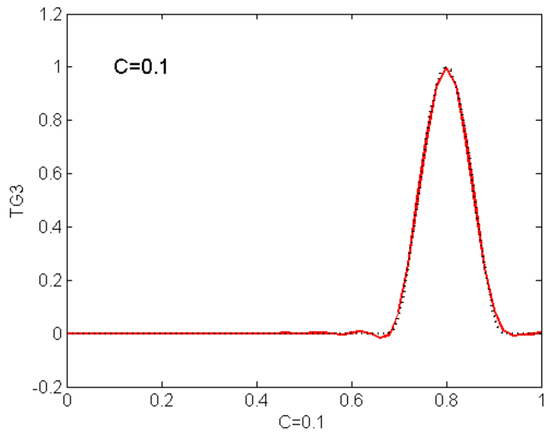

Figure 5. The result of TG3 with C=0.1

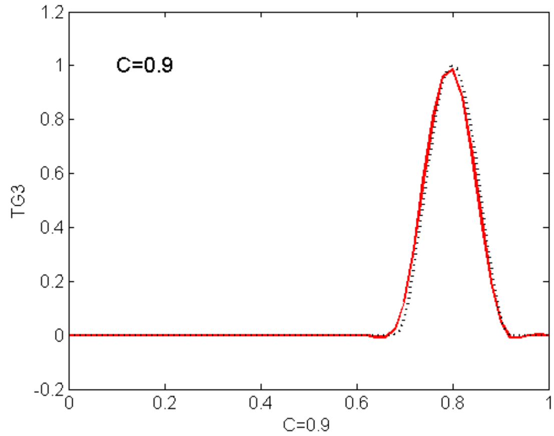

Figure 6. The result of TG3 with C=0.9

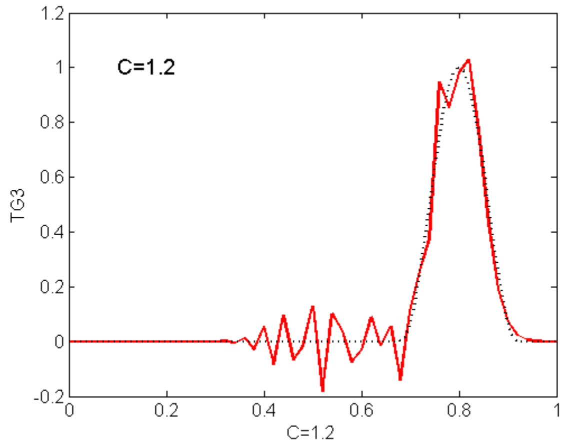

Figure 7. The result of TG3 with C=1.2

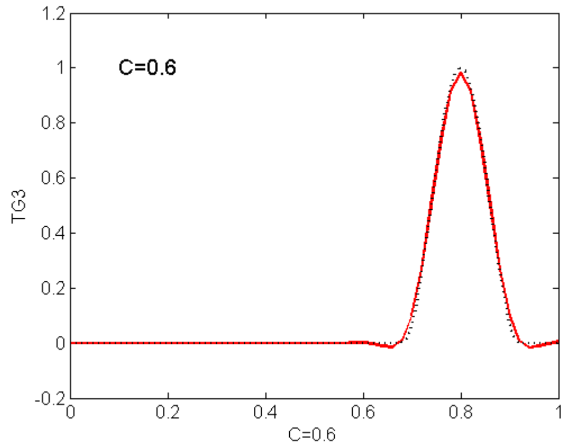

Figure 8. The result of TG3 with C=0.6

The stable condition for TG3 method is $C^2 < 1$. For figure 6, when $C = 1.2$, the results become unstable. On the contrary, the results are stable when $C = 0.1$ and $C = 0.9$. And when $C = 0.1$, the result has less phase error compared to the result we have got when $C = 0.9$.

3. TG4 method

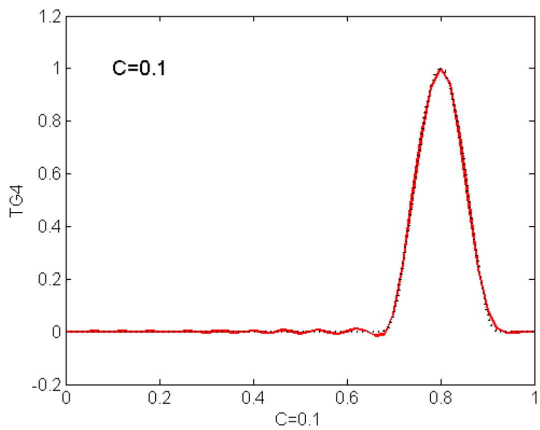

Figure 9. The result of TG4 with C=0.1

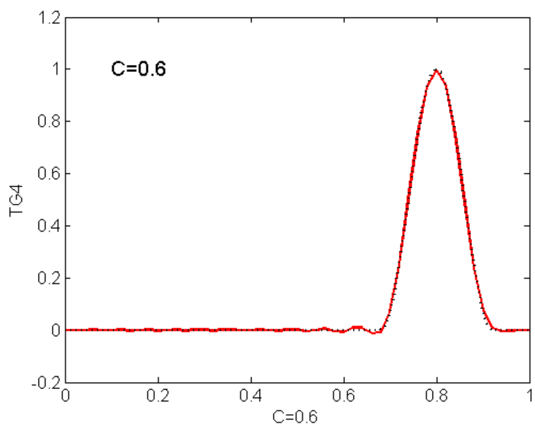

Figure 10. The result of TG4 with C=0.6

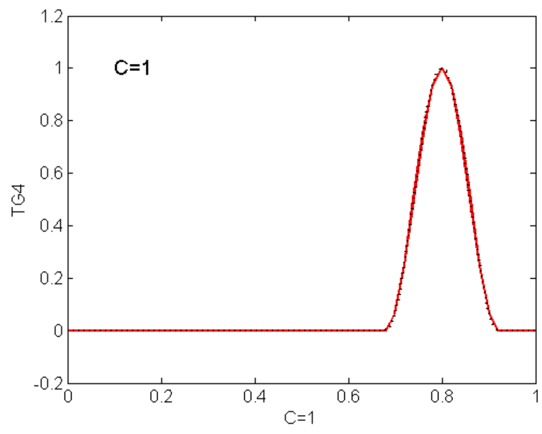

Figure 11. The result of TG4 with C=1

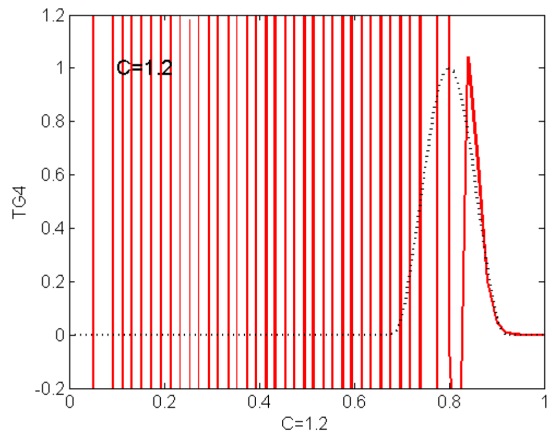

Figure 12. The result of TG4 with C=1.2

From above results, we can find that when $C = 1$, TG4 method works best, there are no oscillations or phase errors.

But if we increase the value of C to 1.2, the result becomes to be unstable. When $C = 0.1$ and $C = 0.6$, the TG4 method works well but rather poor than the result of $C = 1$. And another point is that decreasing the value of C doesn't make the result better.

4. CN method

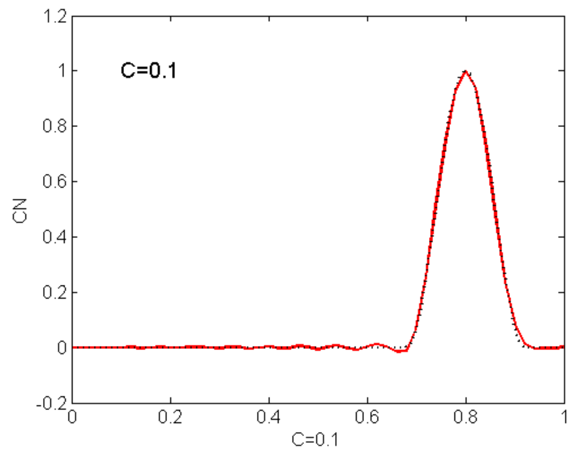

Figure 13. The result of CN with C=0.1

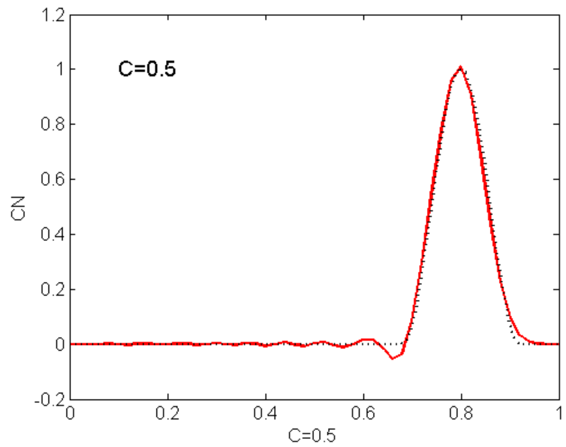

Figure 14. The result of CN with C=0.5

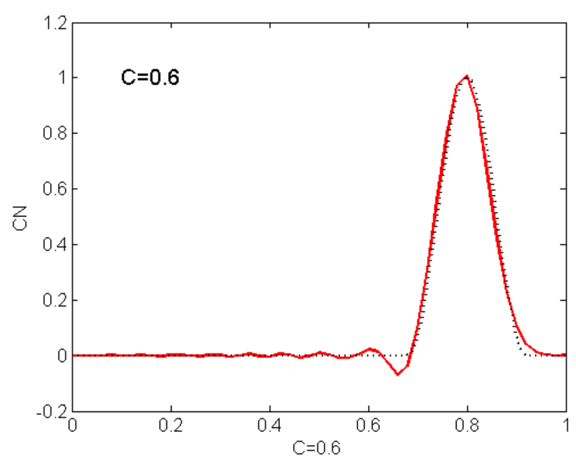

Figure 15. The result of CN with C=0.6

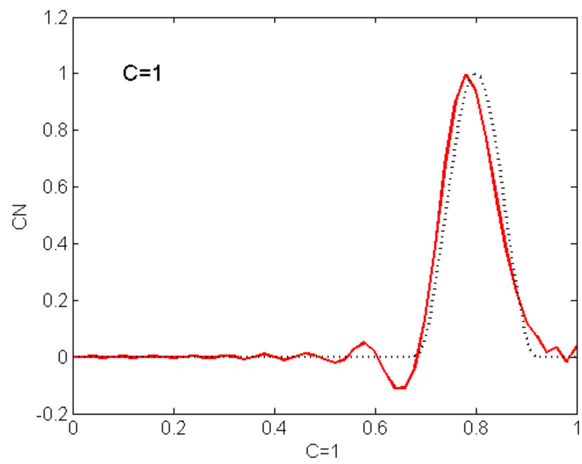

Figure 16. The result of CN with C=1

Even though the CN method is unconditionally stable. But when the value of C is big, the results will show oscillations on the sharp corners of the curve. And the phase error is the reason why this phenomenon happens. By decreasing the value of C, we will make the result become more stable and accurate.

5. The comparison among different methods

When $C = 0.6$, we can find that the TG3 and TG4 work better than CN and TG2. Because when $C = 0.6$, the $C^2 > 1/3$, so TG2 becomes unstable which cause that the result is not accurate. For CN method, the reason is the phase error which makes the results on the sharp corners of the curve inaccurate. And because TG3 and TG4 are both higher order schemes, which avoid the drawbacks of the large phase error, so they are more accurate.

Part II

1. The Courant number

$$C = |a| \cdot \Delta t/h$$

Where $a = 1$, $\Delta x = h = 2 \times 10^{-2}$,$\Delta t = 1.5 \times 10^{-2}$.

So we get:

$$C = 0.75$$

2. CN method and CJ method

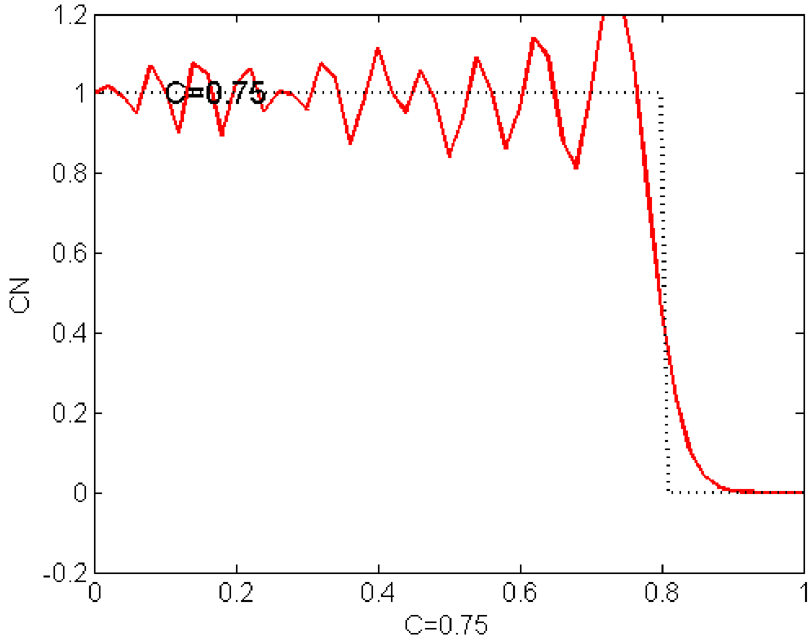

Figure 15. The result of CN with C=0.75

CN method is unconditionally stable. But above result shows oscillations on the domain, which is not accurate. The reason is that the second order schemes could not provide the enough accuracy because of large phase error, so we need to implement higher order method to solve this problem.
In order to solve this problem, we are going to apply the CJ method.

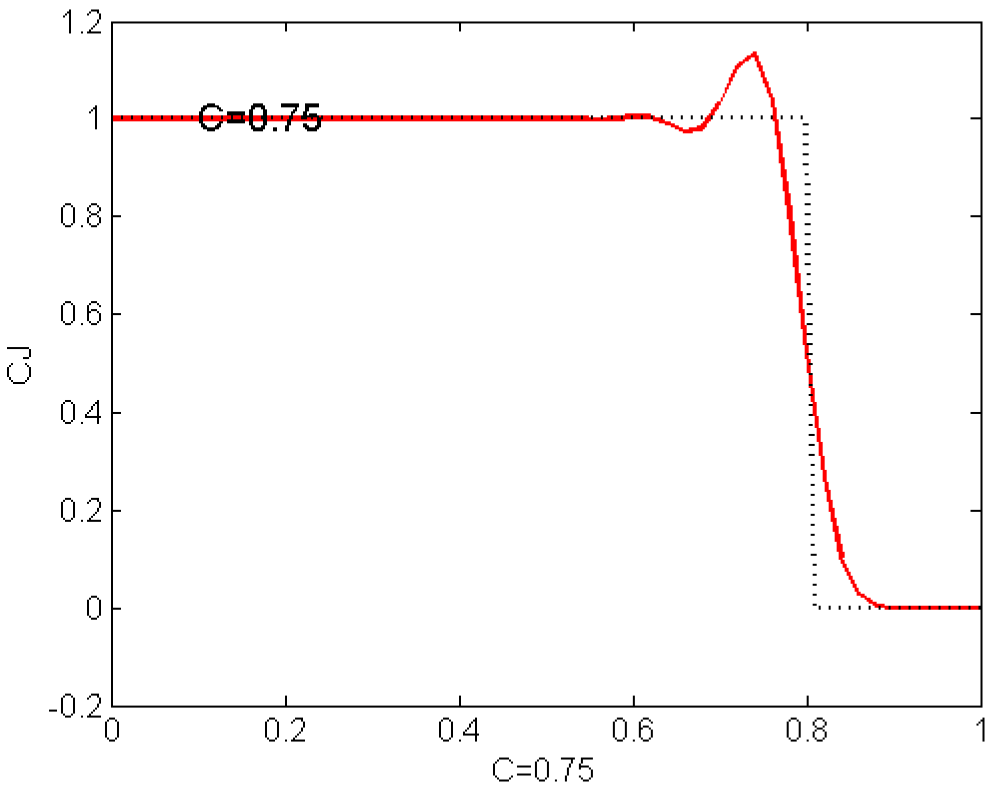

Figure 15. The result of CJ with C=0.75

The result of CJ method is better than the result of CN method. The oscillations are not happening in CJ method. But near the sharp corner of the curve, the result is not accurate.

3. TG2 method and TG2-2S method

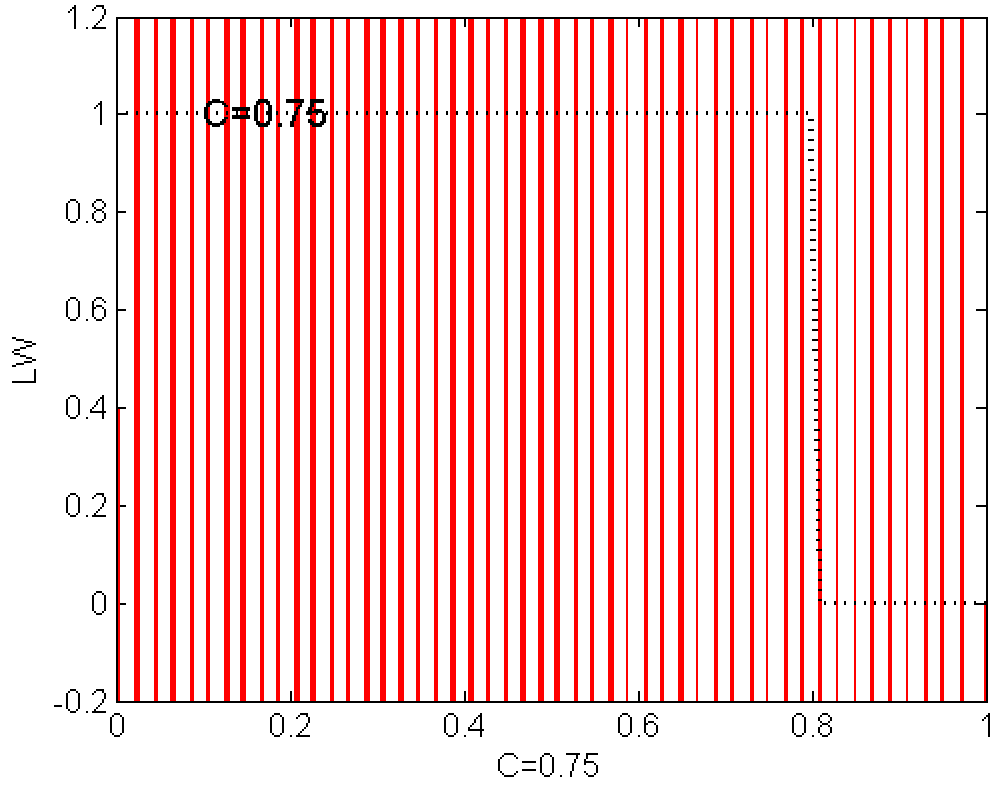

Figure 15. The result of TG2 with C=0.75

The result shows oscillations on the whole domain because the value of C extends the stable condition. For TG2 method, the stable condition is $C^2 < 1/3$. Consequently, the result becomes unstable and oscillates.

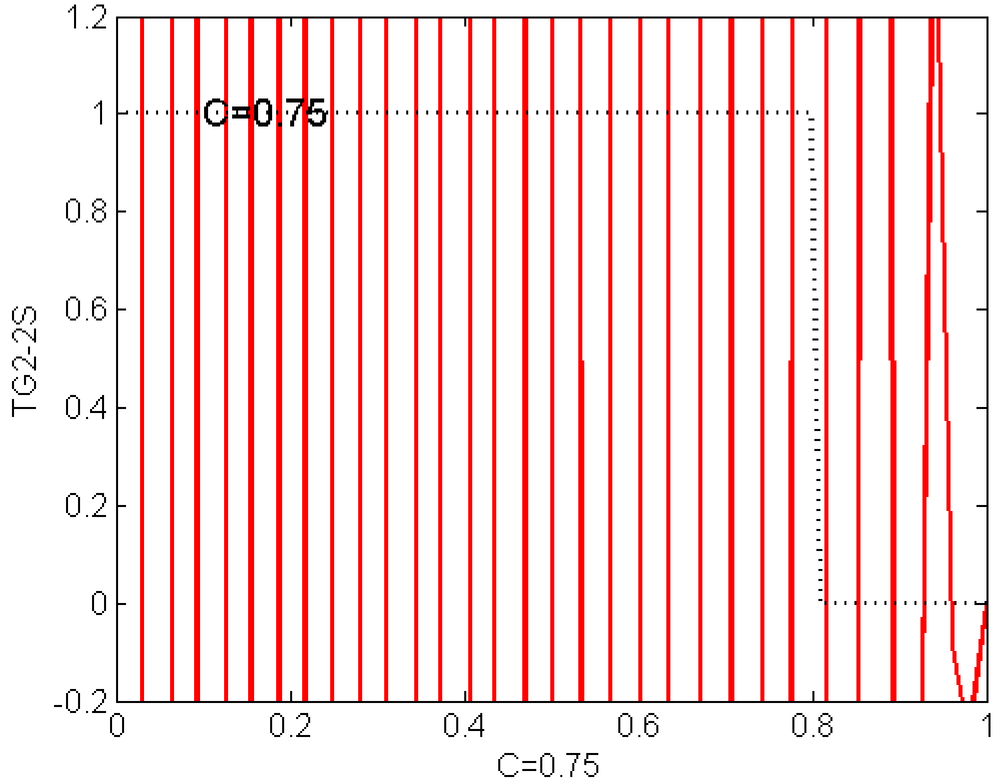

Figure 15. The result of TG2-2S with C=0.75

The result of TG2-2S also oscillates on the whole domain. The reason is same to TG2 method.
As a result, for $C = 0.75$, we need to apply other methods whose stable conditions are higher than this to avoid oscillations.

Part III

The modification of the code

```
A(isp,isp) = A(isp,isp) + w_ig*(N'*N + dt_2*a*(N'*Nx + Nx'*N + dt_2*a*Nx'*Nx));
B(isp,isp) = B(isp,isp) - w_ig*dt*a*(N'*Nx + dt_2*a*Nx'*Nx);
```

Figure 15. The modification of the code of CJ method

```
A1(isp,isp) = A1(isp,isp) + w_ig*(N'*N);
B1(isp,isp) = B1(isp,isp) - w_ig*((dt/2*N'*(a*Nx)));
f1(isp) = f1(isp) + w_ig*(N')*SourceTerm(x);
A2(isp,isp) = A2(isp,isp) + w_ig*(N'*N);
B2(isp,isp) = B2(isp,isp);
f2(isp) = f2(isp) + w_ig*(N')*SourceTerm(x);
C2(isp,isp) = C2(isp,isp) - w_ig*(dt*N'*(a*Nx));
```

Figure 16. The modification of the code of TG2-2S method