FINITE ELEMENTS IN FLUIDS

Assignment 1

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→ REACH ONE FORM OF THE MOMENTUM EQUATION FROM THE OTHER.

$$\rho\frac{\partial \underline{v}}{\partial t} + \rho(\underline{v}\cdot\nabla)\underline{v} - \nabla\cdot\underline{\underline{\sigma}} = \rho b \qquad \text{NON CONSERVATIVE FORM}$$

$$\Updownarrow$$

$$\frac{\partial(\rho\underline{v})}{\partial t} + \nabla\cdot(\rho\underline{v}\otimes\underline{v} - \underline{\underline{\sigma}}) = \rho\underline{b} \qquad \text{CONSERVATIVE FORM}$$

starting from the conservative form:

$$\frac{\partial(\rho\underline{v})}{\partial t} + \nabla\cdot(\rho\underline{v}\otimes\underline{v} - \sigma) = \rho\underline{b}\,,$$ applying the chain rule and the distributive

$$\rho\frac{\partial \underline{v}}{\partial t} + v\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) - \nabla\cdot\sigma = \rho\underline{b}$$

but the divergence of a tensor product follows the identity:

$$\nabla\cdot(\underline{a}\otimes\underline{b}) = \underline{b}(\nabla\cdot\underline{a}) + (\underline{a}\cdot\nabla)\underline{b}$$

thus, $\nabla\cdot(\rho\underline{v}\otimes\underline{v}) = \underline{v}(\nabla\cdot(\rho\underline{v})) + (\rho\underline{v}\cdot\nabla)\underline{v}$

substituting on the momentum equation

$$\rho\frac{\partial \underline{v}}{\partial t} + \underline{v}\frac{\partial \rho}{\partial t} + \underline{v}(\nabla\cdot(\rho\underline{v})) + (\rho\underline{v}\cdot\nabla)\underline{v} - \nabla\cdot\underline{\underline{\sigma}} = \rho\underline{b}$$

$$\rho\frac{\partial \underline{v}}{\partial t} + \underline{v}\underbrace{\left(\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{v})\right)}_{\text{mass conservation } = 0} + \rho(\underline{v}\cdot\nabla)\underline{v} - \nabla\cdot\underline{\underline{\sigma}} = \rho\underline{b}$$

$$\boxed{\rho\frac{\partial \underline{v}}{\partial t} + \rho(\underline{v}\cdot\nabla)\underline{v} - \nabla\cdot\underline{\underline{\sigma}} = \rho\underline{b}}$$ non conservative form of the momentum equation