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HW # 1

I'm watching you

Starting from the non Conservation form of the momentum equation

$$\rho \frac{\partial \bar{u}}{\partial t} + \rho(\bar{u}.\nabla)\bar{u} = \rho\bar{b} + \nabla.\underline{\sigma}$$

using ①

$$\frac{\partial \rho\bar{u}}{\partial t} - \bar{u}\frac{\partial \rho}{\partial t} + \rho(\bar{u}.\nabla)\bar{u} = \rho\bar{b} + \nabla.\underline{\sigma}$$

using ②

$$\frac{\partial \rho\bar{u}}{\partial t} - \bar{u}\frac{\partial \rho}{\partial t} + \nabla.(\rho\bar{u}\otimes\bar{u}) - \bar{u}\,\nabla.(\rho\bar{u}) = \rho\bar{b} + \nabla.\underline{\sigma}$$

$$\frac{\partial \rho\bar{u}}{\partial t} = \rho\bar{b} + \bar{u}\left(\cancel{\frac{\partial \rho}{\partial t} + \nabla.(\rho\bar{u})}\right) + \nabla.(\underline{\sigma} - \rho\bar{u}\otimes\bar{u})$$

→ 0 mass Conservation

$$\boxed{\frac{\partial(\rho\bar{u})}{\partial t} = \rho\bar{b} + \nabla.(\underline{\sigma} - \rho\bar{u}\otimes\bar{u})}$$ The Conservative form

$$\frac{\partial \rho\bar{u}}{\partial t} = \rho\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \rho}{\partial t}$$

$$\rho\frac{\partial \bar{u}}{\partial t} = \frac{\partial \rho\bar{u}}{\partial t} - \bar{u}\frac{\partial \rho}{\partial t} \rightarrow ①$$

$$\nabla.(\rho\bar{u}\otimes\bar{u}) = \bar{u}\,\nabla.(\rho\bar{u}) + \rho(\bar{u}.\nabla)\bar{u}$$

$$\rho(\bar{u}.\nabla)\bar{u} = \nabla.(\rho\bar{u}\otimes\bar{u}) - \bar{u}\,\nabla.(\rho\bar{u}) \rightarrow ②$$

Same principles Can be applied to go from the Conservative to non Conservative form.