HOMEWORK 2: PURE CONVECTION

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1 1D PURE CONVECTION

In this assignment we solve the following equation and boundary conditions

$$u_t + au_x = 0 \qquad x\epsilon(0,1) \qquad t\epsilon(0,0.6]$$
$$u(x,0) = u_0(x)n \qquad x\epsilon(0,1)$$
$$u(0,t) = 1 \qquad t\epsilon(0,0.6]$$

where

$$x_0(x) = \begin{cases} 1 & \text{if } x \leq 0.2, \\ 0 & \text{otherwise.} \end{cases}$$

$$a = 1, \Delta x = 2 \cdot 10^{-2}, \Delta t = 1.5 \cdot 10^{-2}$$

This problem has is include in the script `mai.m` as problem 4. To compute the lumped mass matrix we add an output named `Ml` and add some lines as followed

```
M,Ml,K,C = FEM_matrices(X,T,referenceElement)
...
Mel(1,1)= 2*w_ig*N_ig(1);
Mel(2,2)= 2*w_ig*N_ig(2);
```

In the script `System.m` we added the Lax-Wendroff and third-order explicit Taylor-Galerkin (TG3) methods

```
case 1 % Lax-Wendroff + Galerkin
A = M;
B = -dt*a*(C+a*dt/2*K);
methodName = 'L-W';
case 2 % Lax-Wendroff with lumped mass matrix + Galerkin
A = Ml;
B = -dt*a*(C+a*dt/2*K);
methodName = 'L-W FD';
case 3 % Crank-Nicolson + Galerkin
A = M + 1/2*a*dt*C;
```

```
B = -a*dt*C;
methodName = 'CN';
case 4 % Crank-Nicolson with lumped mass matrix + Galerkin
A = Ml + 1/2*a*dt*C;
B = -a*dt*C;
methodName = 'CN FD';
case 5 % TG3 explicit
A = M + 1/6*a^2*dt^2*K;
B = -dt*a*(C+a*dt/2*K);
methodName = 'TG3';
```

After some modifications to the Matlab scripts we resolve the problem. For this problem the value of Courant number is 0.75. The figure 1 shows the results obtained with Crank Nicholson method with a consistent and lumped mass matrix, respectively. We observe that the use of lumped mass matrix decrease the accuracy.

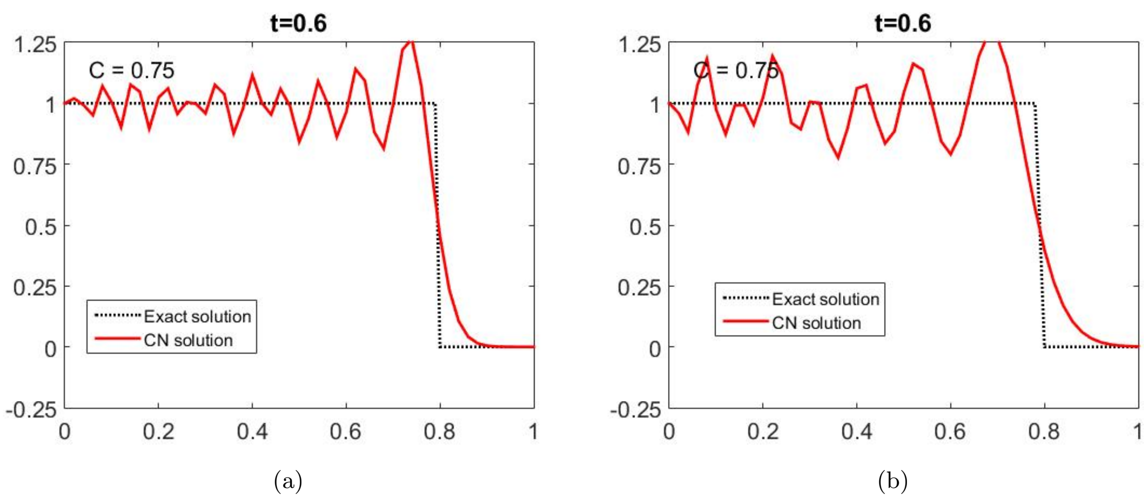
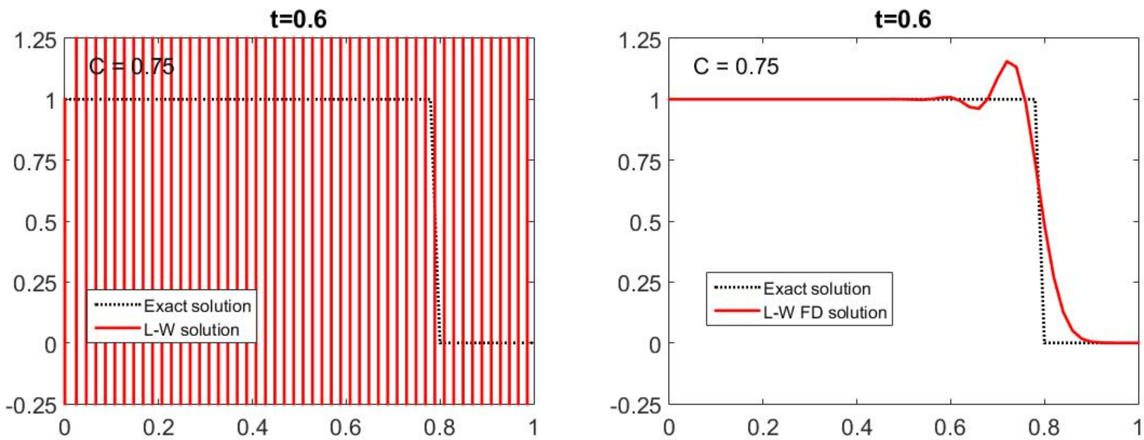


Figure 1: Crank Nicholson method. (a) Consistent mass matrix. (b) Lumped mass matrix.

The figure 2 presents the results of Lax-Wendroff method. For this method the use of lumped mass matrix gives stability. Using the consistent mass matrix is unstable due the value of Courant number. The Courant number must satisfy the following condition $C^2 \leqslant 1/3$.

The solution with TG3 method is shown in figure 3. This method has the best accuracy than the other two methods.

Finally, to observe the influence of Courant number we reduce the time discretization with 60 time steps ($\Delta t = 0.01$) and compute a lower value $C = 0.5$. The figure 4 present the results of the three methods. The Lax-Wendroff method increase the accuracy significantly due to the method became stable with $C = 0.5$. The other methods do not have a notorious improvement.



(a) (b)

Figure 2: Lax-Wendroff method. (a) Consistent mass matrix. (b) Lumped mass matrix.

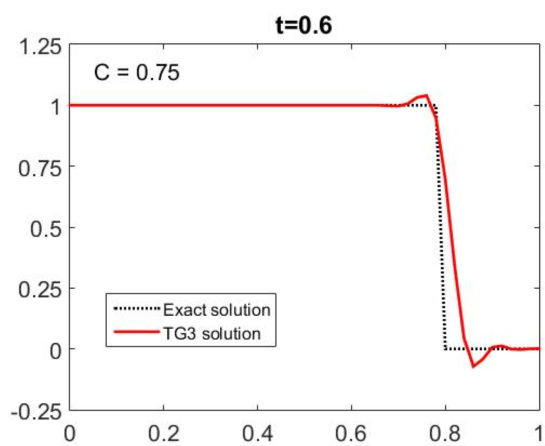


Figure 3: Crank Nicholson

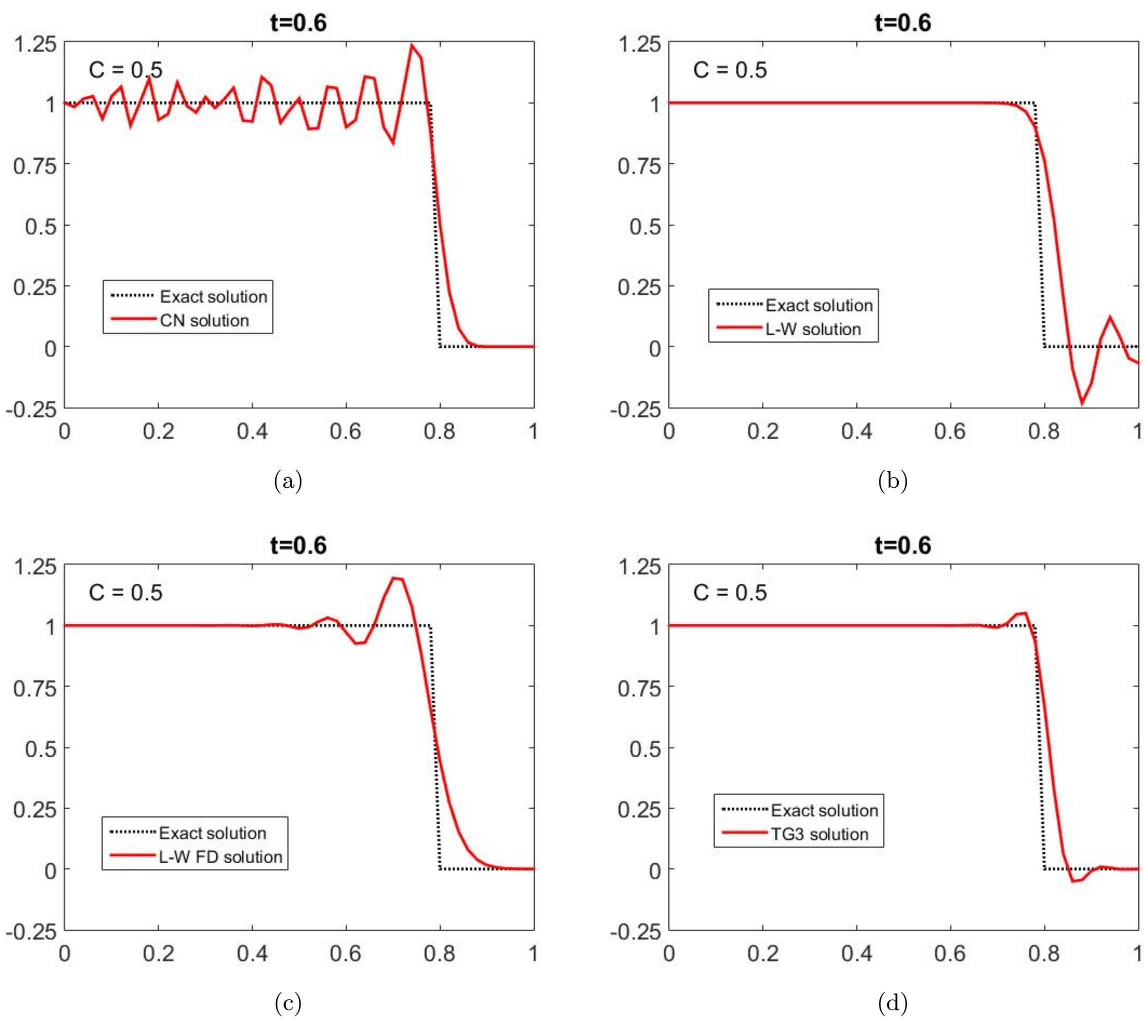


Figure 4: Reduced time discretization $\Delta t = 0.01$ (a) CN method. (b) Lax-Wendroff. (c) Lax-Wendroff with lumped mass matrix. (c) TG3.

2 Conclusions

Courant number is an indicator of the stability of the solution and is dependent of the space and time discretization. Reducing the value of Courant number will help to the stability of Lax Wendroff and TG3 methods.

The use of lumped mass matrix increase the accuracy of Lax-Wendroff method. However, Crank-Nicholson method decrease the accuracy using the lumped mass matrix.

TG3 method is the more accurate of the three employed methods.