Assignatura

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DNI

$$u_t^\varepsilon + u^\varepsilon u_x^\varepsilon = \varepsilon u_{xx}^\varepsilon \qquad (x,t) \in [-1,1] \times [0,T]$$

$$u^\varepsilon(x,0) = u_0(x) \quad \text{for } x \in [-1,1]$$

$$u^\varepsilon(-1,t) = u^\varepsilon(1,t) = 0 \quad \text{for } t \in [0,T]$$

(Will drop the superscript $\varepsilon$ from $u^\varepsilon$)

<u>one-step Taylor Galerkin:</u>

$$f_x = u u_x - \varepsilon u_{xx}$$

$$f = \frac{1}{2}(u^2) - \varepsilon u_x$$

$$u^{n+1} = u^n + \Delta t\, u_t + \frac{1}{2}(\Delta t)^2 u_{tt}$$

$$u_t = \frac{u^{n+1} - u^n}{\Delta t} - \frac{1}{2}\Delta t\, u_{tt}$$

from the equation we have

$$u_t = \varepsilon u_{xx} - u u_x = -f_x(u)$$

$$u_{tt} = -f_{xt} = -(a u_t)_x = (a f_x)_x \quad \text{where } a = \frac{\partial f}{\partial u}$$

Substituting in the differential equation

$$\boxed{\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{2}\Delta t\,(a f_x)_x - f_x(u^n)} \quad \text{where } f_x = u u_x - \varepsilon u_{xx}$$

<u>two-step Taylor Galerkin:</u>

$$u^{n+\frac{1}{2}} = u^n + \frac{\Delta t}{2} u_t^n$$

$$u^{n+1} = u^n + \Delta t\, u_t^{n+1/2}$$

substitute in the differential equation

$$\boxed{\begin{aligned} u^{n+\frac{1}{2}} &= u^n - \frac{\Delta t}{2} f_x(u^n) \\ u^{n+1} &= u^n - \Delta t\, f_x(u^{n+\frac{1}{2}}) \end{aligned}}$$

done on two steps

$$f_x = u u_x - \varepsilon u_{xx}$$

good approximation for $f$ should be made (high order shape functions should be chosen)

$$f(u) \simeq \sum_1^n N_i(x) f(u_i)$$