$u^\varepsilon_t + u^\varepsilon u^\varepsilon_x = \varepsilon u^\varepsilon_{xx}$ || $(x,t) \in [-1,1] \times [0,T]$

$u^\varepsilon(x,0) = u_0(x)$ || for $x \in [-1,1]$

$u^\varepsilon(-1,t) = u^\varepsilon(1,t) = 0$ || for $t \in [0,T]$

* Using a one step Taylor Galerkin

$f_x = u^\varepsilon u^\varepsilon_x - \varepsilon u^\varepsilon_{xx}$

$f = \frac{1}{2}(u^\varepsilon)^2 - \varepsilon u^\varepsilon_x$

$\rightarrow (u^\varepsilon)^{n+1} = (u^\varepsilon)^n + \Delta t\, u^\varepsilon_t + \frac{1}{2}\Delta t^2 u^\varepsilon_{tt}$

$u^\varepsilon_t = \dfrac{(u^\varepsilon)^{n+1} - (u^\varepsilon)^n}{\Delta t} - \frac{1}{2}\Delta t\, u^\varepsilon_{tt}$

$u^\varepsilon_t = \varepsilon u^\varepsilon_{xx} - u^\varepsilon u^\varepsilon_x = -f_x(u^\varepsilon)$

$u^\varepsilon_{tt} = -f_{xt} = -(a u^\varepsilon_t)_x = (a f_x)_x$ || $a = \dfrac{df}{du}$

$\rightarrow$ Substitute in the dif. eq.

$\dfrac{(u^\varepsilon)^{n+1} - (u^\varepsilon)^n}{\Delta t} = \frac{1}{2}\Delta t (a f_x)_x - f_x (u^\varepsilon)^n$ || $f_x = u^\varepsilon u^\varepsilon_x - \varepsilon u^\varepsilon_{xx}$

* Using two step Galerkin

$(u^\varepsilon)^{n+1/2} = (u^\varepsilon)^n + \frac{\Delta t}{2}(u^\varepsilon_t)^n$

$(u^\varepsilon)^{n+1} = u^n + \Delta t\, u_t^{n+1/2}$

$\rightarrow$ Substitute in the diff. eq.

$(u^\varepsilon)^{n+1/2} = (u^\varepsilon)^n - \frac{\Delta t}{2} f_x(u^\varepsilon)^n$

$(u^\varepsilon)^{n+1} = (u^\varepsilon)^n - \Delta t\, f_x(u^\varepsilon)^{n+1/2}$