Finite Elements in Fluids **March 20, 2019**
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Assignment IV:

Propagation of a steep front
The unsteady-convection equation governing the propagation of a steep front is the following:

$$\begin{cases} u_t + au_x = 0 & x \;\epsilon\; (0,1), t \;\epsilon\; (0, 0.6] \\ u(x,0) = u_0 & x \;\epsilon\; (0,1) \\ u(0,t) = 1 & t \;\epsilon\; (0, 0.6] \end{cases}$$

$$u_0(x) = \begin{cases} 1 & \text{if } x \leq 0.2 \\ 0 & \text{otherwise} \end{cases}$$

Where: $a = 1$, $\Delta x = 2 \cdot 10^{-2}$, $\Delta t = 1,5 \cdot 10^{-2}$

1 Courant Number

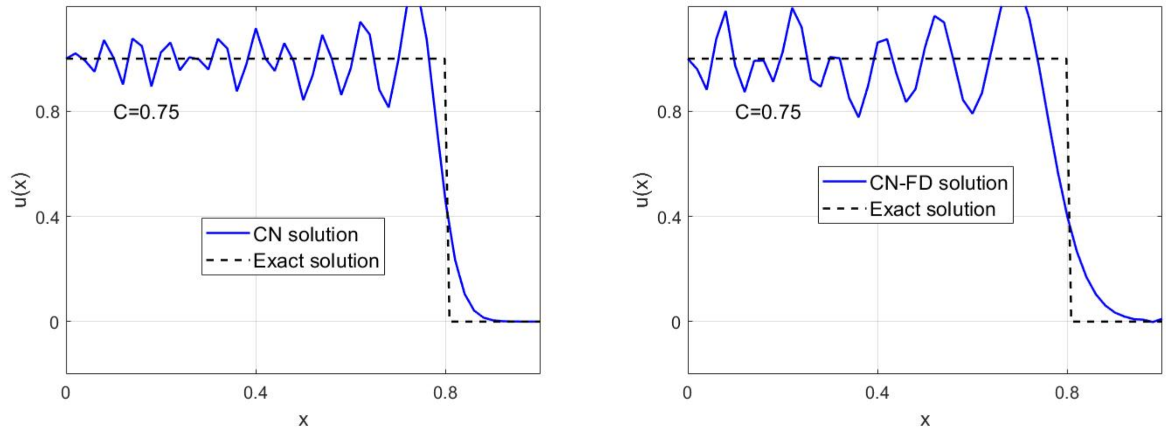
The courant number is defined as:

$$C = \frac{|a|\Delta t}{h} \tag{1}$$

Thus for this case the Courant number is:

$$C = \frac{1 \cdot 1,5 \cdot 10^{-2}}{2 \cdot 10^{-2}} = 0,75 \tag{2}$$

2 Solve the problem using the Crank-Nicholson scheme in time and linear finite element for the Galerkin scheme in space. Is the solution accurate?



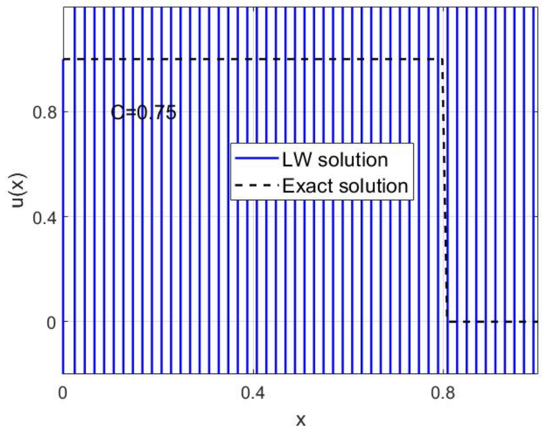
(a) Consistent mass matrix

(b) Lumped mass matrix

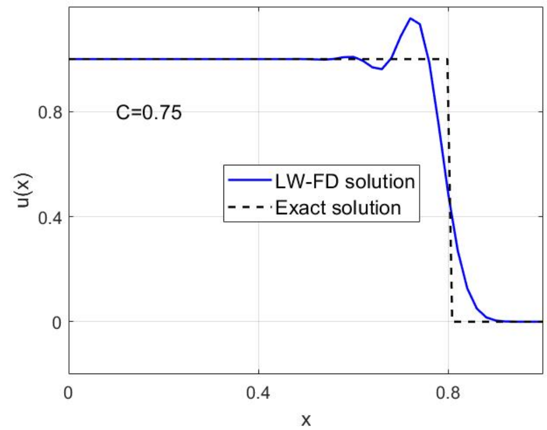
Figure 2.1: Crank-Nicolson implementation

As you can see, the results are more accurate when a consistent mass matrix is used as shown in figure (a) and there is an error is greater if a lumped mass matrix is used as shown in figure (b).
However, both solutions are stable.

3 Solve the problem using the second-order Lax-Wendroff method. Can we expect the solution to be accurate? If not, what changes are necessary? Comment the results.



(a) Consistent mass matrix



(b) Lumped mass matrix

Figure 3.1: Law-Wendroff Galerkin implementation (TG2)

When $C \geq \frac{\sqrt{3}}{3}$ the TG2 method shows serious instabilities as it is possible to see it in the image (a) for a consistent mass matrix. But, as it is possible to see in image (b), the stability range expands when a lumped mass matrix is used and this can be observed since the Courant number is the same in both cases.

4 Solve the problem using the third-order explicit Taylor-Galerkin method. Comment the results.

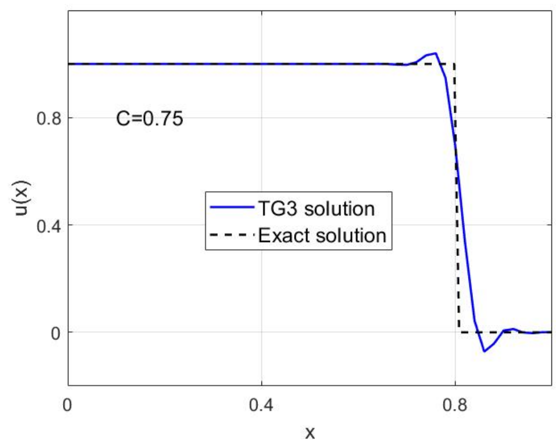


Figure 4.1: Third-order Taylor Galerkin implementation (TG3)

The solution by the TGR method can be observed in figure 4.1.
The method has a third order precision in time and therefore the error is lower in comparison with TG2 and CN.
Also, since C is less than 1, the scheme is stable.