Incompressible Flow

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Ques 1. Stokes Problem.

In the Virtual Learning Centre, you can find a Matlab code for solving a Stokes problem with analytical solution.

a. Write a Matlab function to compute the velocity and pressure errors

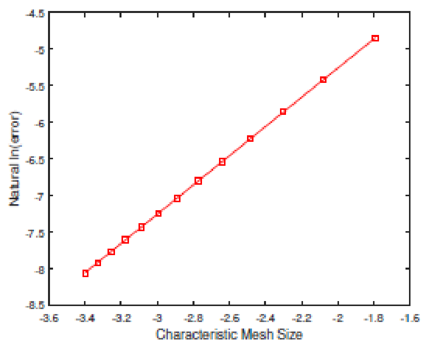
$$e_v = \sqrt{\int (\nabla u - \nabla u^h)^2 \, d\Omega} \quad \& \; e_p = \sqrt{\int (p - p^h)^2 \, d\Omega}$$

where (u; p) is the exact solution and $(u^h; p^h)$ the numerical solution. Check the convergence for Q2Q1 and Q2Q0 elements. Do the errors behave as expected?

b. Solve the problem using P1P1 elements with an stabilised formulation. Check the convergence of this approximation.

Solution:

In section 1.1, velocity and pressure errors are computed by modifying the given algorithm. The convergence for Q2Q1 & Q2Q0 element computed. The following figures shows the error computed for different mesh size and number of elements.

<table>
<tr><th>Element Q2Q1</th><th>Element Q2Q0</th></tr>
<tr><td>


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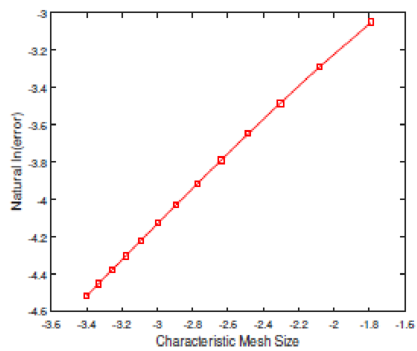

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<tr><td>Fig: Velocity Error – Mesh Size</td><td>Fig: Velocity Error- Mesh Size</td></tr>
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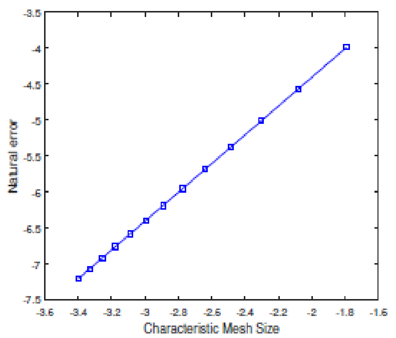

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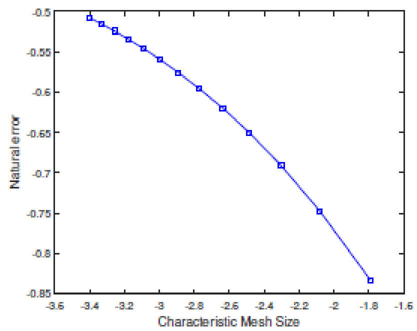

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<tr><td>Fig: Pressure Error- Mesh Size</td><td>Fig: Pressure Error- Mesh Size</td></tr>
</table>

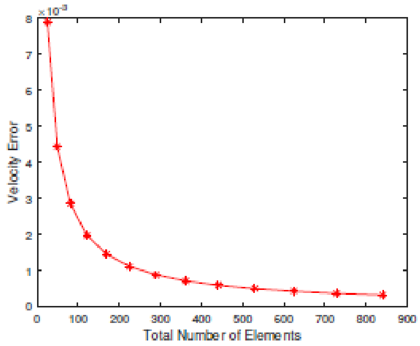

Fig: Velocity Error- No. of Elements

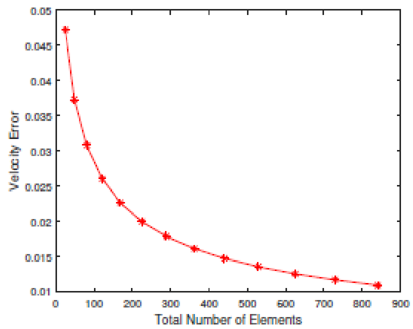

Fig: Velocity Error- No. of Elements

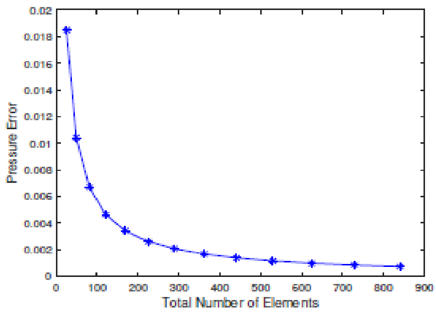

Fig: Pressure Error- No. of Elements

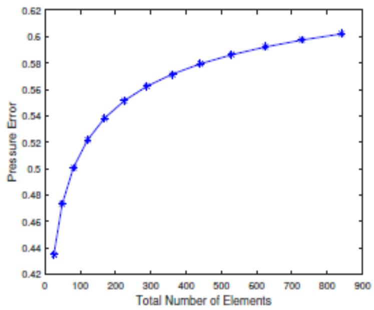

Fig: Pressure Error- No. of Elements

Error seen in the above figure is as expected for both the elements. Velocity error decreases as no of element increases for both the elements. Pressure error decreases as mesh size increases for Q2Q1 element and exactly opposite is observed in Q2Q0 element.

Q2Q1 element converges fast compare to Q2Q0 element for increase in number of elements and Q2Q0 elements shows unexpected increase in pressure error for different mesh size because it fails to capture accurate pressure behaviour.

In section 1.2, P1P1 element i.e., linear triangular mesh is represented. P1P1 element doesn't satisfy LBB condition.

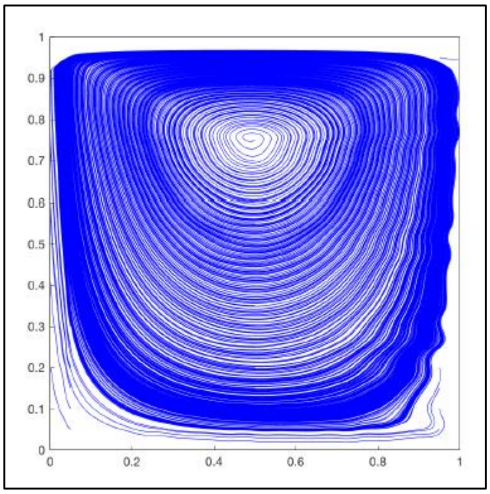

Fig: Streamline

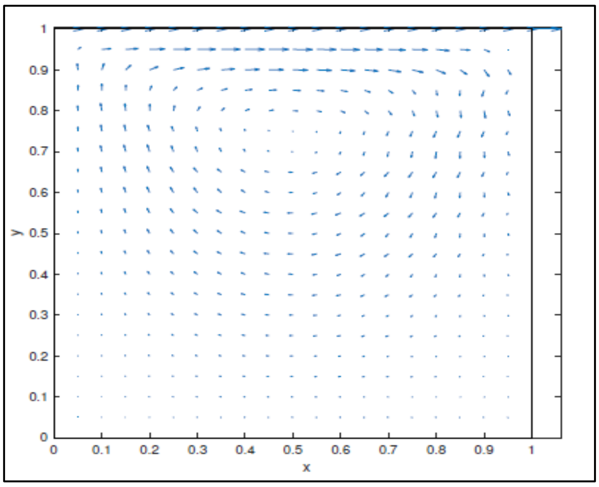

Fig: Velocity Field

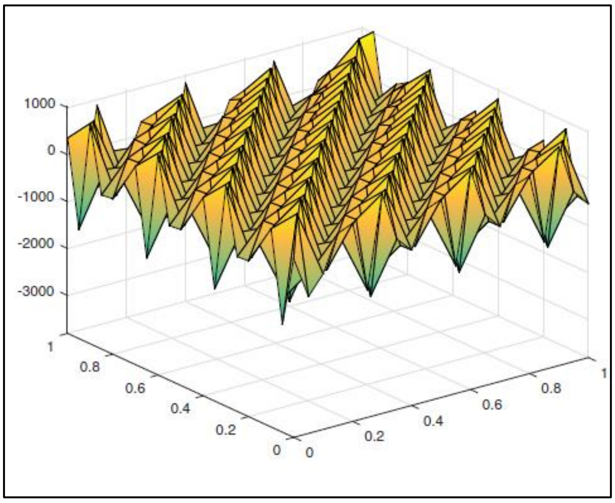

Fig: Pressure Field

It is observed from the figure that pressure field shows node to node spurious oscillations. Same can be observed in the streamlines near boundary mesh.

Ques 2. Cavity Flow

In the Virtual Learning Center, you can find a Matlab code for solving the Cavity flow problem

Compute the solution of the Stokes problem considering (i) a structured, uniform mesh of Q2Q1 elements with 20 elements per side (ii) a structured mesh of 20x20 Q2Q1 elements refined near the walls. Comment on the results. Describe the main properties of the velocity and pressure fields. Are there any differences between the solutions obtained with these two meshes? Which one do you think is the best? Why?

Solution:

i. Structured Uniform Mesh of 20X20 Q2Q1 elements:

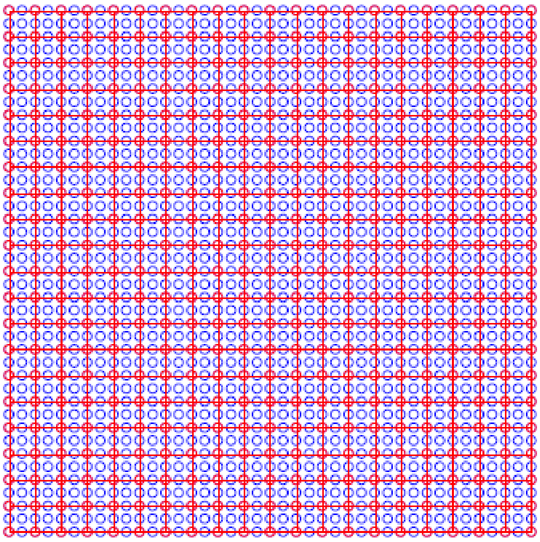

Fig: Mesh

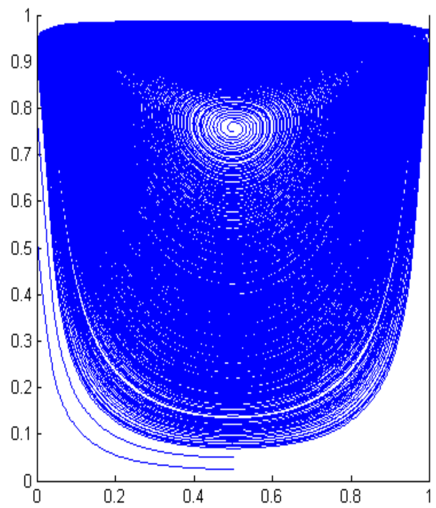

Fig: Streamline

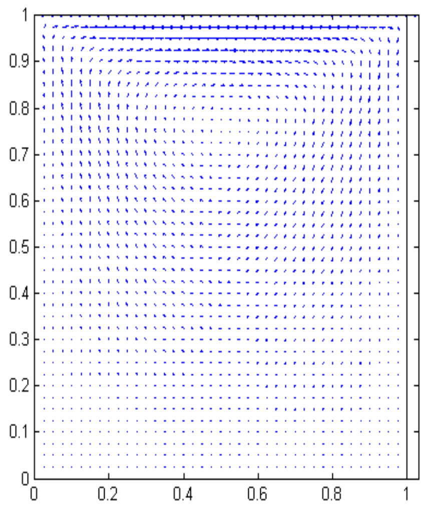

Fig: Velocity Field

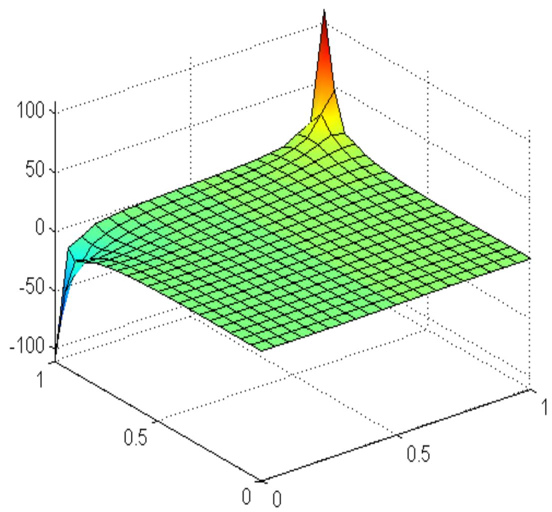

Fig: Pressure Field

It is observed from the figures that it is quadratic converging. Streamlines shows complete symmetry of the solution. Pressure field behaves extremely accurate as it satisfies LBB condition.

ii. Structured Mesh of 20X20 Q2Q1 elements refined near walls:

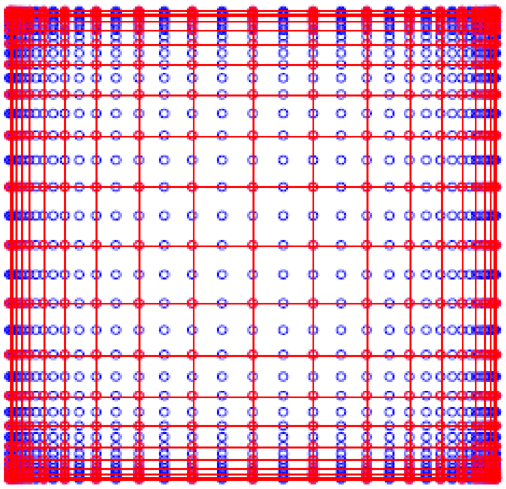

Fig: Refined Mesh

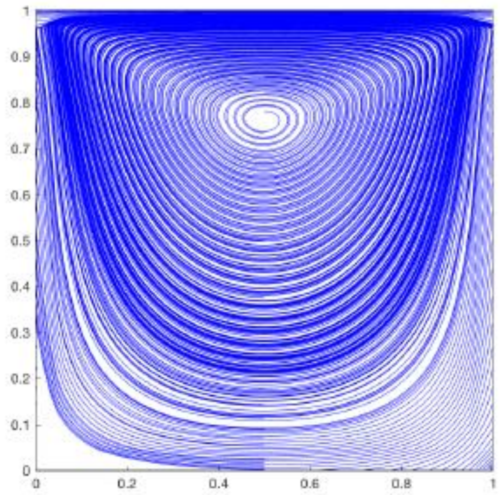

Fig: Streamlines

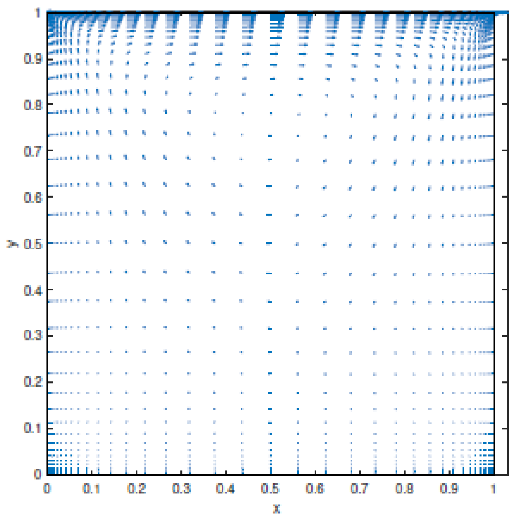

Fig: Velocity Field

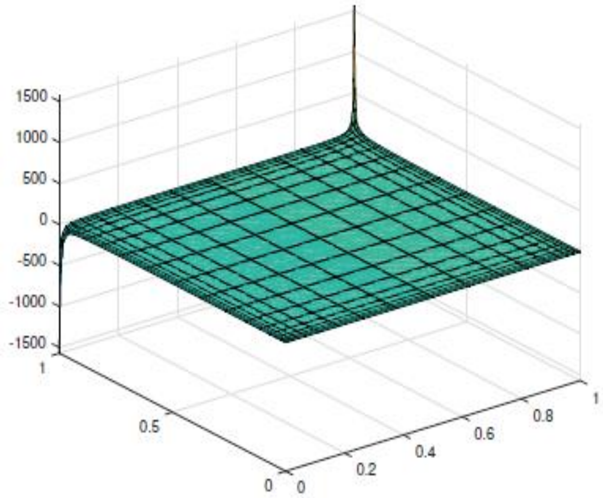

Fig: Pressure Field

It is observed from the figure that refined mesh shows no impact on streamlines, they are same as of uniform mesh. Velocity field is more concentrated near the wall and pressure field shows smoother transition from zero to non-zero values of pressure.

-----x-----x-----x-----