Finite element in fluid

Assignment 2

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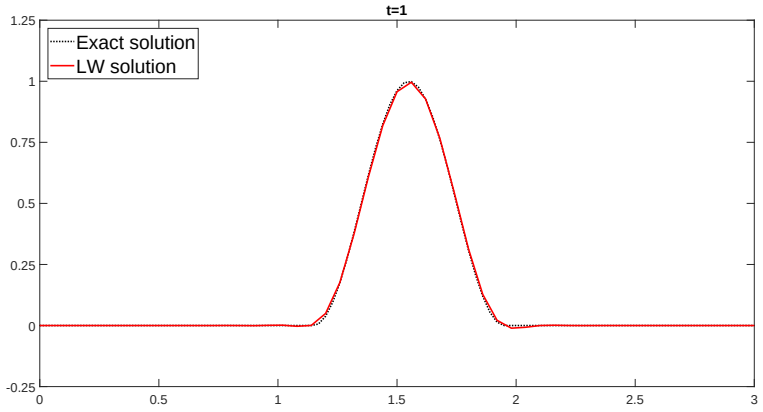

Figure 1: lax wandroff

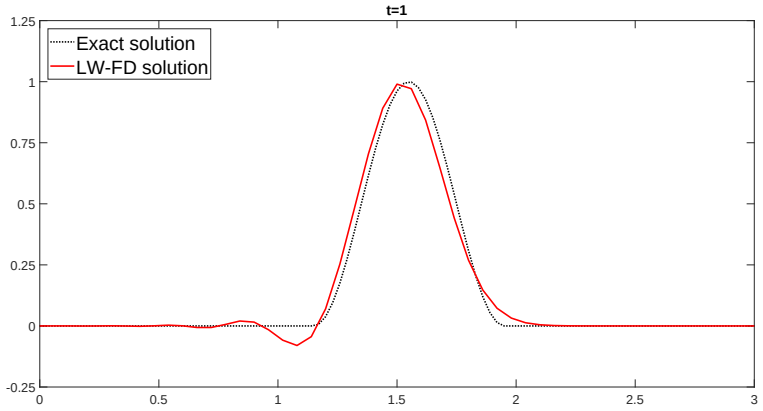

Figure 2: lax wandroff with lumped mass matrix

Courant number = 0.25

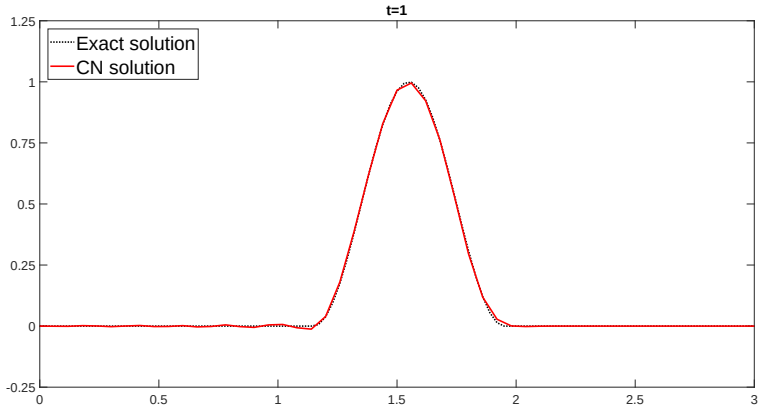

Figure 3: Crank Niclson

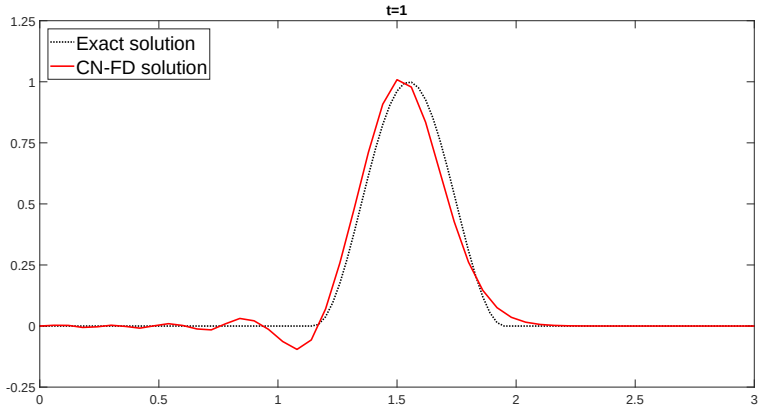

Figure 4: Crank Niclson with lumped mass matrix

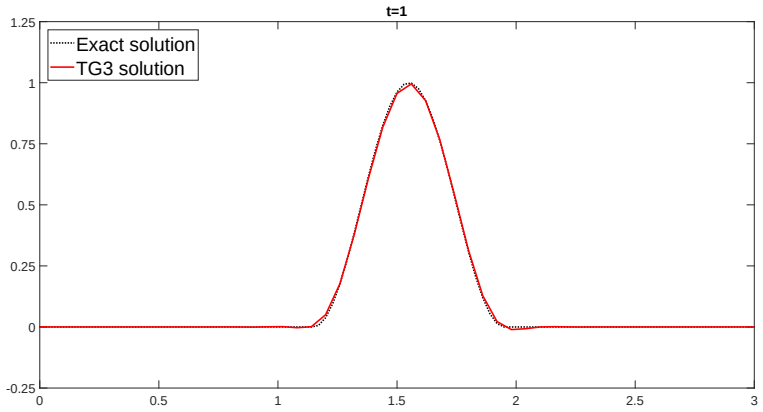

Figure 5: TG3

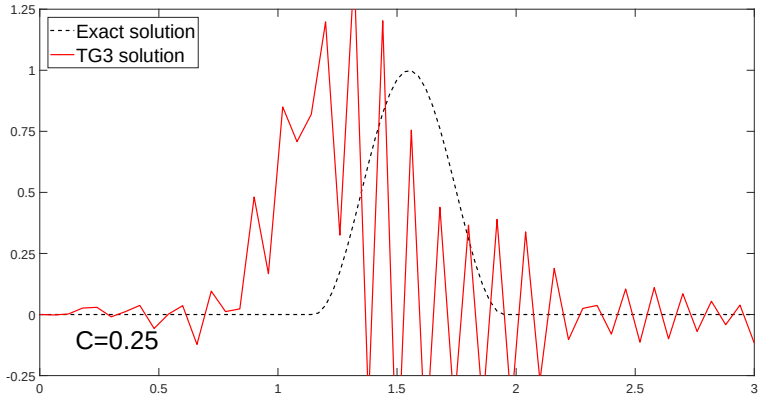

Figure 6: Leap frog

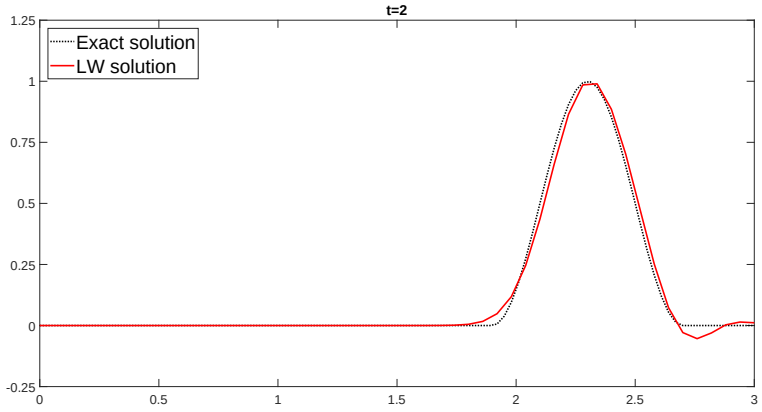

Figure 7: lax wandroff

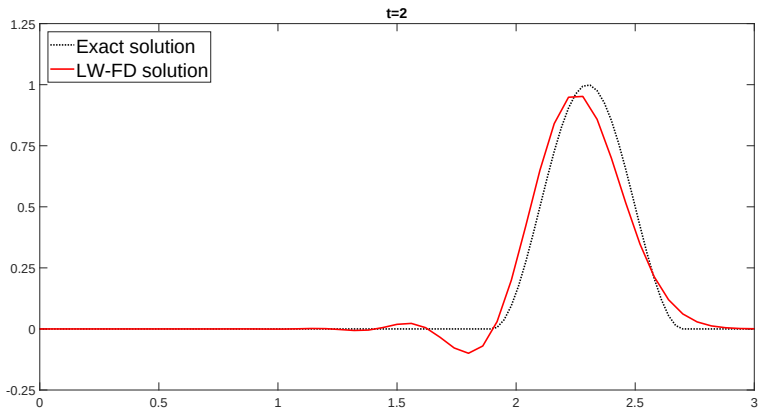

Figure 8: lax wandroff with lumped mass matrix

Courant number = 0.5

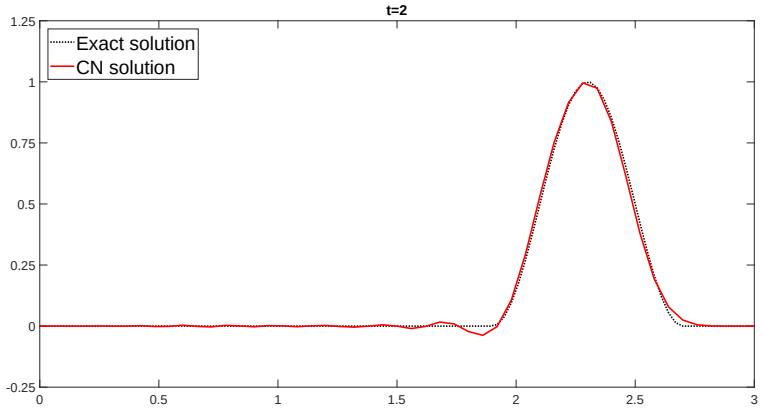

Figure 9: Crank Niclson

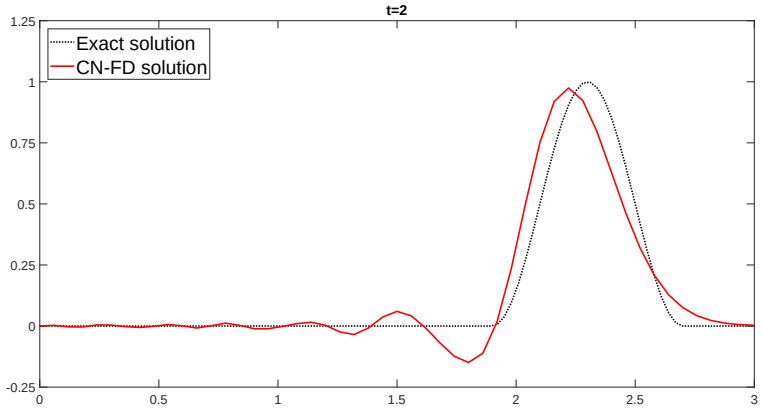

Figure 10: Crank Niclson with lumped mass matrix

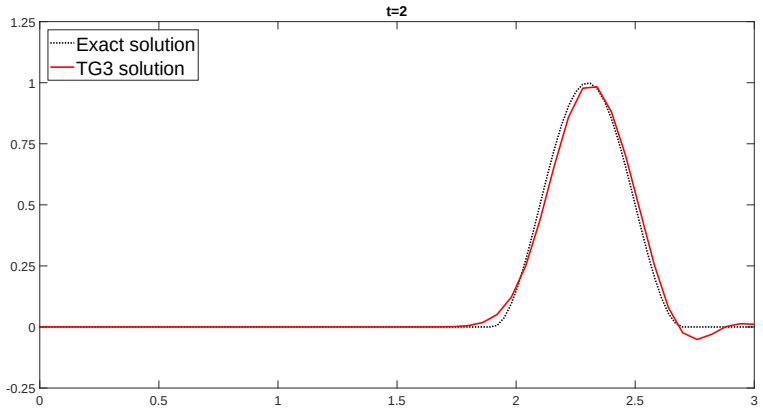

Figure 11: TG3

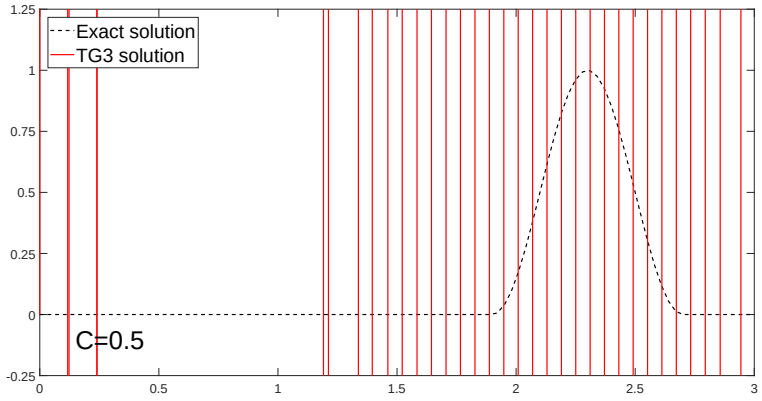

Figure 12: Leap frog

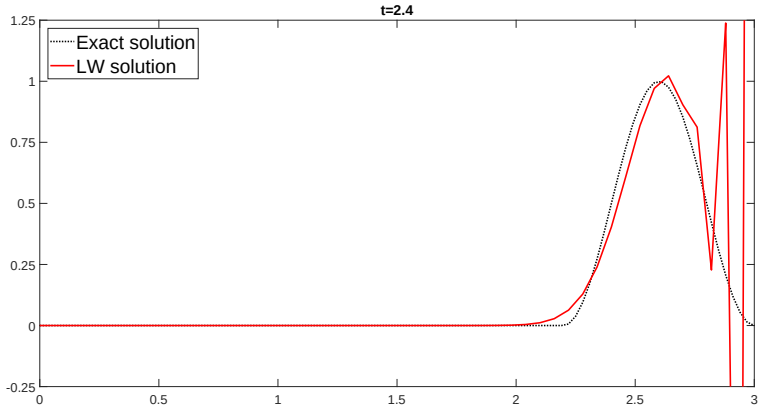

Figure 13: lax wandroff

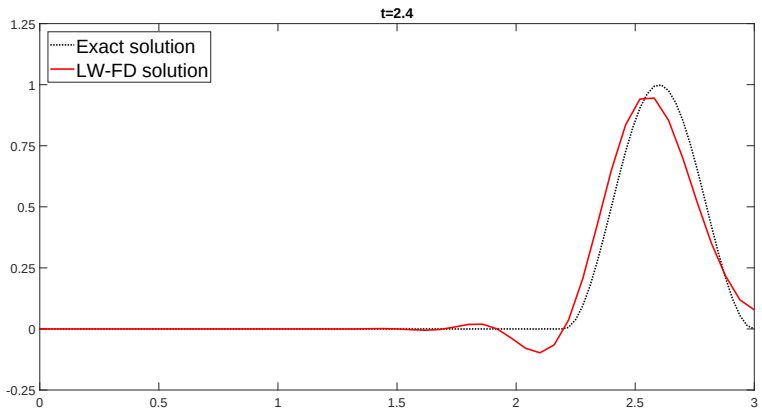

Figure 14: lax wandroff with lumped mass matrix

Courant number = 0.6

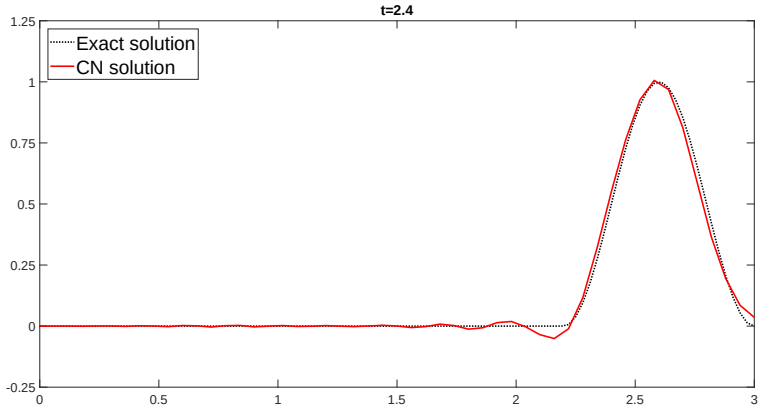

Figure 15: Crank Niclson

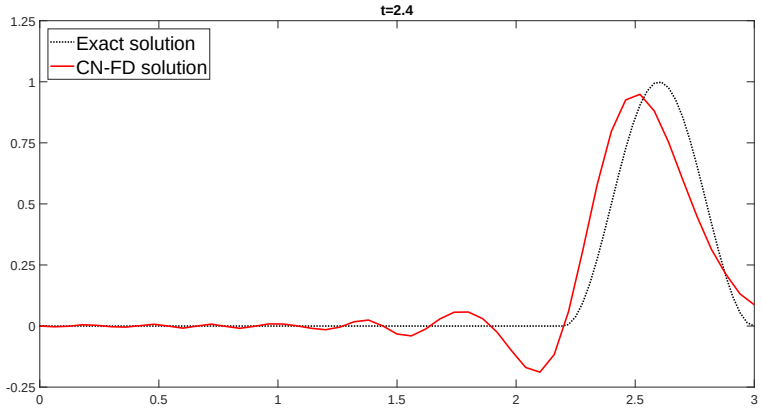

Figure 16: Crank Niclson with lumped mass matrix

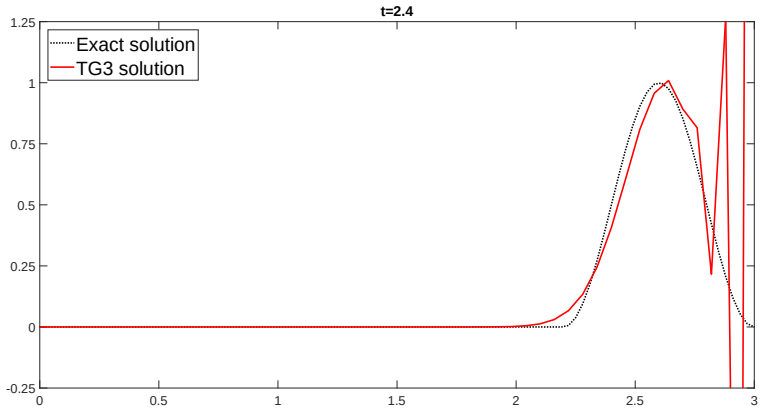

Figure 17: TG3

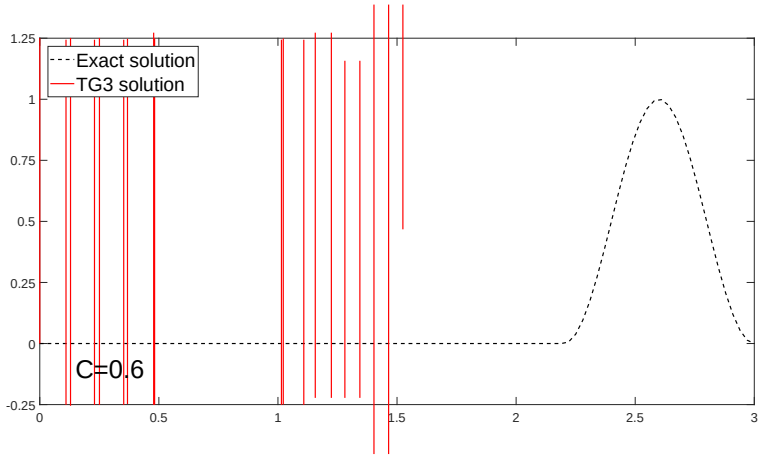

Figure 18: Leap frog

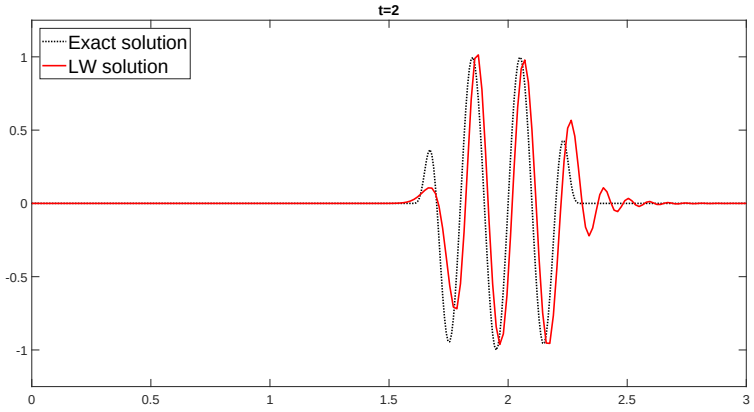

Figure 19: lax wandroff

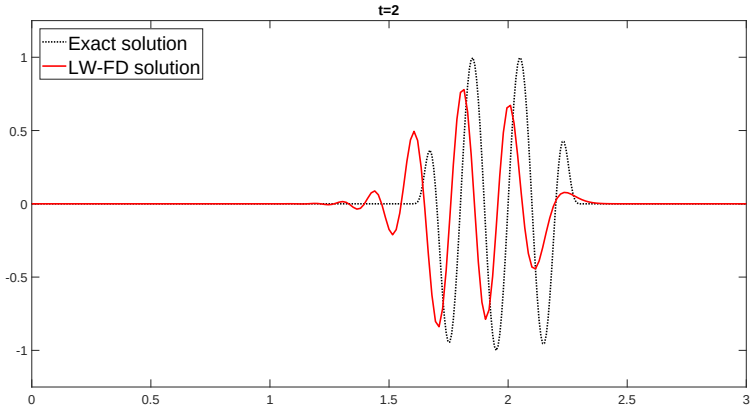

Figure 20: lax wandroff with lumped mass matrix

Courant number = 0.5

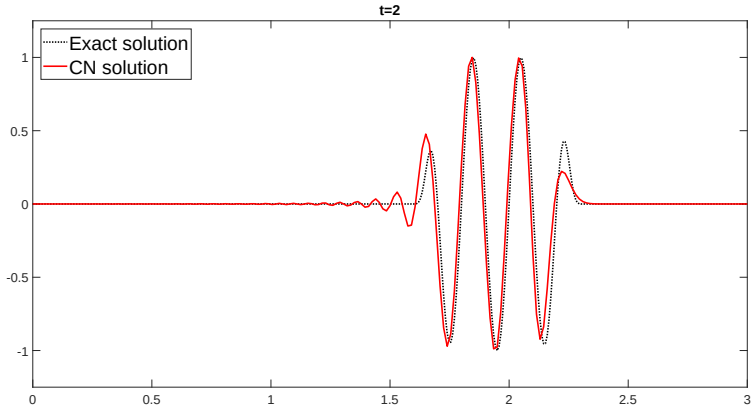

Figure 21: Crank Niclson

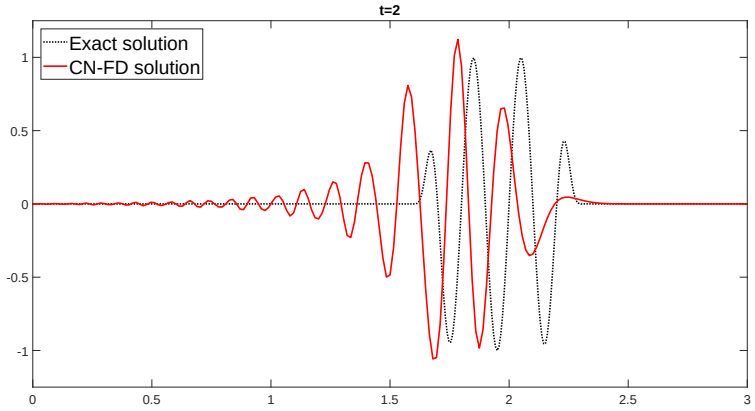

Figure 22: Crank Niclson with lumped mass matrix

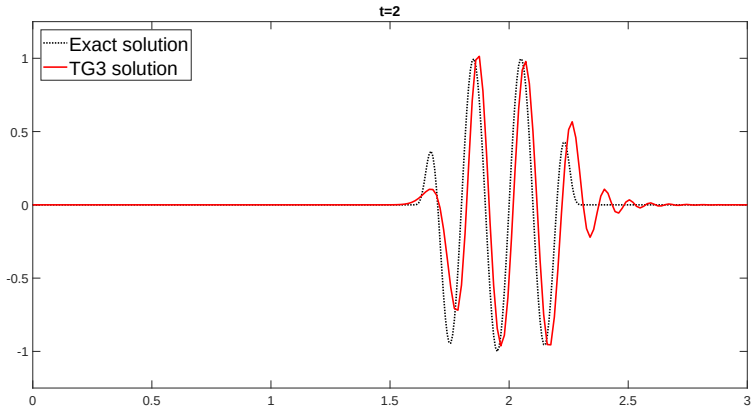

Figure 23: TG3

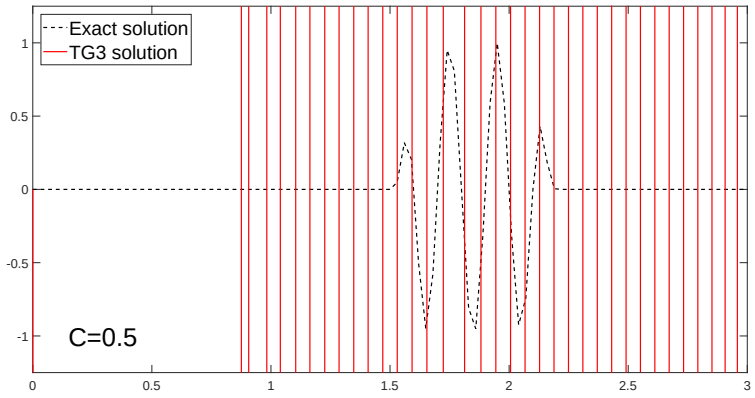

Figure 24: Leap frog

For Lax wendroff and TG3 using a consistent mass matrix show a better phase accuracy than Lax wandroff schemes combined with diagonal mass matrix and Lax wandroff consistent mass cannot be operated with $C^2 > \frac{1}{3}$. Phase error affect Lax wendroff with consistent mass matrix and Leap frog. TG3 has rather uniform phase accuracy over the entire stable interval $0 < c < 1$ of the courant number.

The schemes using Finite elements with consistent mass matrix shows a superior phase accuracy.
The diagonal mass matrix can reduce computational cost and it can increase the stability range and less accuracy in the stability region.
Crank Nicolson consistent finite element shows a superior phase accuracy and it increases with the time step size.
Crank Nicolson consistent finite element and Crank Nicolson diagonal mass matrix show large phase error.
TG3 method shows very accurate results because of it is high order time accuracy.
For large values of courant number show maximum accuracy.
Implicit method shows an excellent phase response size at different values of courant number.

Annex

case 1 Lax-Wendroff + Galerkin
$A = M$;
$B = -a * dt * C - 0.5 * a^2 * dt^2 * K$;
methodName = 'LW';

case 2 Lax-Wendroff with lumped mass matrix + Galerkin
$A = diag(sum(M))$;
$B = -a * dt * C - 0.5 * a^2 * dt^2 * K$;
methodName = 'LW-FD';

case 3 Crank-Nicolson + Galerkin
$A = M + 1/2 * a * dt * C$;
$B = -a * dt * C$;
methodName = 'CN';

case 4 Crank-Nicolson with lumped mass matrix + Galerkin
$A = diag(sum(M)) + 1/2 * a * dt * C$;
$B = -a * dt * C$;
methodName = 'CN-FD';

case 5 Third order Taylor-Galerkin + Galerkin
$A = M + (1/6) * dt^2 * a^2$;
$B = -a * C * dt - 0.5 * a^2 * dt^2 * K$;
methodName = 'TG3';

case 6 Leap-frog method
$A = M$;
$B = -a * 2 * dt * C$;
methodName = 'LF';